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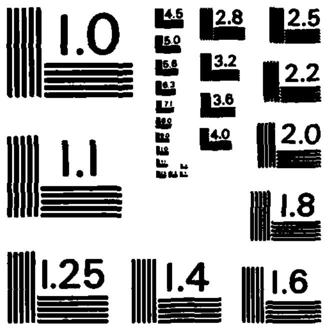
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AFWAL-TR-82-1124

STRAPDOWN TECHNOLOGY DEVELOPMENT

JAY N. BURNS
REFERENCE SYSTEMS BRANCH
SYSTEM AVIONICS DIVISION

SEPTEMBER 1982

Final Report for Period 1 October 1978 to 1 October 1982

Approved for public release; distribution unlimited.

AVIONICS LABORATORY
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This technical report has been reviewed and is approved for publication.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This effort was intended to provide an opportunity for becoming familiar with strapdown inertial navigation hardware and the software necessary for alignment and navigation. To that end a Honeywell H-478 inertial sensor assembly was selected for experimentation. A significant portion of the effort was involved with designing and building an interface unit to permit outputs to be processed on the PDP 11/40 computer. An alignment computer program was written to provide coarse and fine alignment using

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H-478 sensor output data. Also the output data was used to drive a strapdown navigation computer model.

FOREWORD

This report was written by Jay N. Burns of the Reference Systems Branch, Systems Avionics Division, Avionics Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio.

The work documented here was carried out under Project Work Unit 60951519. This work was accomplished during the period 1 Oct 1978 to 1 Oct 1981. The documentation was prepared during CFY 82.

The author wishes to acknowledge the special contributions of certain individuals over the course of the effort: Sgt. Tom Adams, who, with Lt. J. A. Carretto, designed and constructed the H-478 interface unit; Lt. John W. Weiser, who operated the H-478 and obtained experimental data as well as prepared routines for plotting the final results of alignment and navigation; Mr. J. E. Barnes, who prepared and executed computer routines for analyzing, plotting and preparing the H-478 data for use in the alignment and navigation routines; Mr. S. H. Musick, whose help in preparing the analytic basis for the alignment routine was invaluable. Finally, the author wishes to thank Ms. Elizabeth Ditmer for preparing the printed form in which this report now appears, using a computer program named RUNOFF.

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FIGURES

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1.0 INTRODUCTION

This effort was initiated to provide an opportunity to become familiar with strapdown inertial navigation hardware and the software necessary for alignment and navigation. To that end, a Honeywell H-478 inertial sensor assembly was selected for experimentation. The H-478 ISA is a simplified strapdown inertial system which is small, lightweight and easy to use and maintain. The gyros and accelerometers mounted in the H-478 ISA sense angular rates and specific force about three body axes. Each output is digitized to form attitude change (delta theta) and velocity change (delta vee) pulses. The H-478 ISA was mounted on the precision alignment table in the stationary evaluation laboratory (SEL), and the outputs were delivered to the PDP 11/40 computer for processing and recording. In order for the PDP 11/40 to receive the H-478 output signals, it was necessary to provide an appropriate interface unit. This was designed and built in an in-house effort which represented a significant portion of the total effort. The interface is a self-contained unit which permits the monitoring of the H-478 signals and provides centralized power requirements.

The gyro and accelerometer outputs from the H-478 ISA were recorded on 9-track tape and supplied as inputs to alignment and navigation computer programs hosted on the Control Data Corporation (CDC) 6600 computer.

The alignment program consisted of a coarse alignment phase followed by a three-state Kalman filter which executed the fine alignment. Plots of alignment attitude - heading, pitch, and roll and their errors - were obtained as functions of time.

The navigation program was the Draper INSS program, which was modified to accept the H-478 gyro and accelerometer data from tape. Based on these inputs and the orientation of the platform, found from the alignment mode, position, velocity and attitude were computed as a function of time. The differences between computed and actual values, i.e. errors, were plotted for horizontal position and velocity.

2.0 H-478 INTERFACE

The H-478 interface is a self-contained, multifunction unit designed to collect and format signals from the H-478 Inertial Sensor Assembly (ISA) for use on the PDP-11/40 microcomputer. The interface contains its own power supply and cooling systems. Each one of the data lines from the H-478 INS can be monitored by use of front panel mounted test points. Frequencies can be checked by an attached counter.

The unit was designed for easy troubleshooting and maintenance. To reduce ground loops, all circuitry is isolated from ground except at one point, and each chip is individually connected to Vcc and Gnd. The unit also has power control functions for the H-478.

The interface circuitry is basically made up of three sections; the polarity phasing and decode logic, the control/interrupt controller, and the data storage/output section.

2.1 Polarity Phasing and Decode Logic

The polarity phasing and decode logic are shown in Figure 2-1. The ISA gyro and accelerometer output voltages are

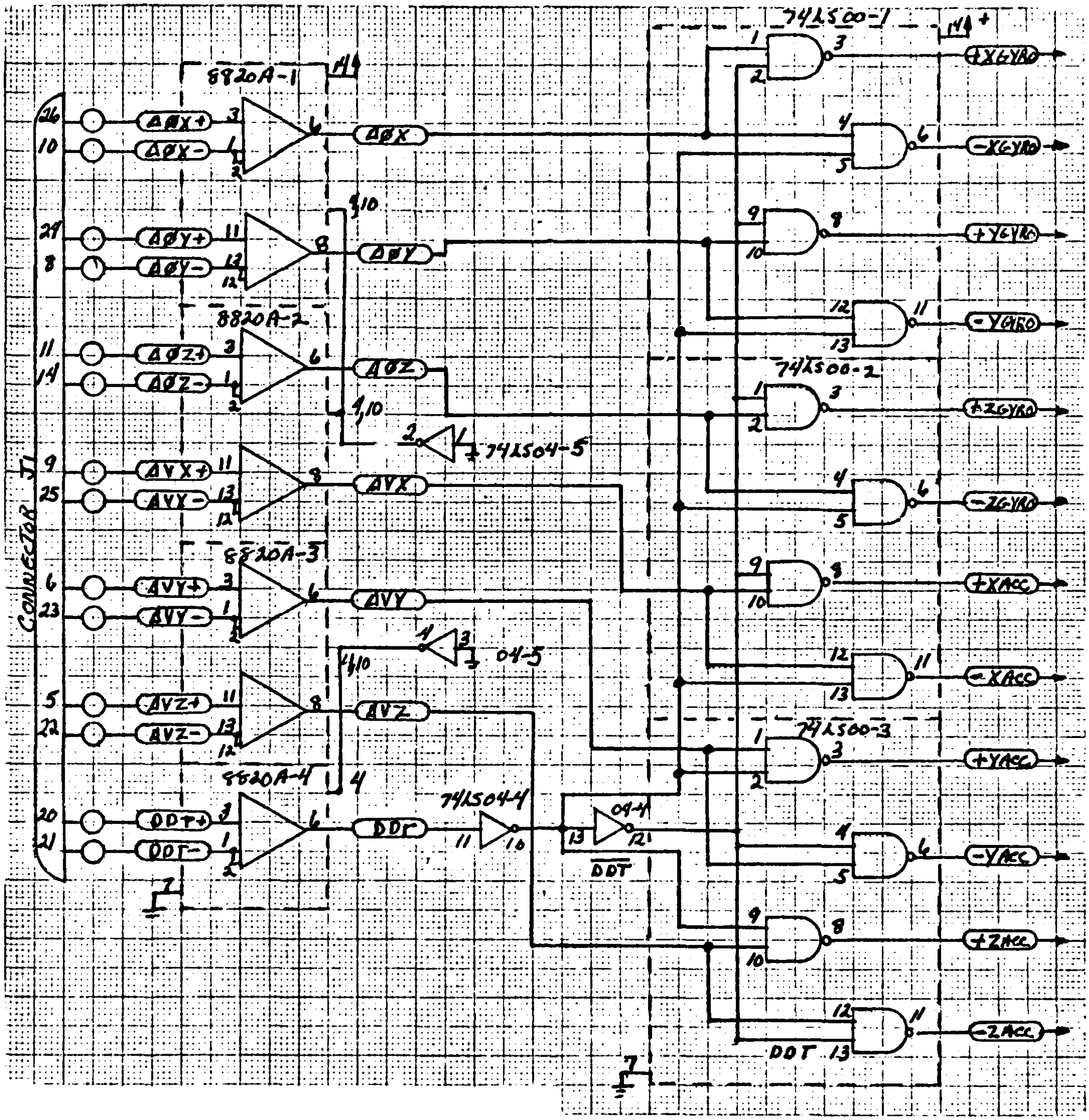


FIGURE 2.1 POLARITY PHASING AND DECODE LOGIC

fed into integrators to be digitized into delta theta and delta vee output pulses. Pulse trains are provided by transistor-transistor-logic (TTL) compatible differential line drivers which are polarity encoded with reference to a common 12.8 KHz square wave decode signal diode-transistor-logic (DDT). The output of the ISA are HI and LO and pulses and a HI and LO digital decode reference signal (DDT). The polarities are determined by encoding the and signals with the DTL pulse. The HI and LO refer to the fact that the signals are from differential amplifiers, providing both a positive and a negative signal.

The polarity phasing and decode logic section is made up of DS8820A dual line receivers and 74LS00 dual input NAND gates. The output is a pair of TTL compatible data signals, one for "positive" pulses, and one for "negative" pulses. These are then fed to the data storage and output section.

2.2 Control/Interrupt Controller

The control/interrupt controller is shown in Figure 2-2. The term "data cycle", as used in this explanation, indicates the transfer of data from the interface into computer memory. The number of data cycles per second is determined by the computer program and can be changed. The

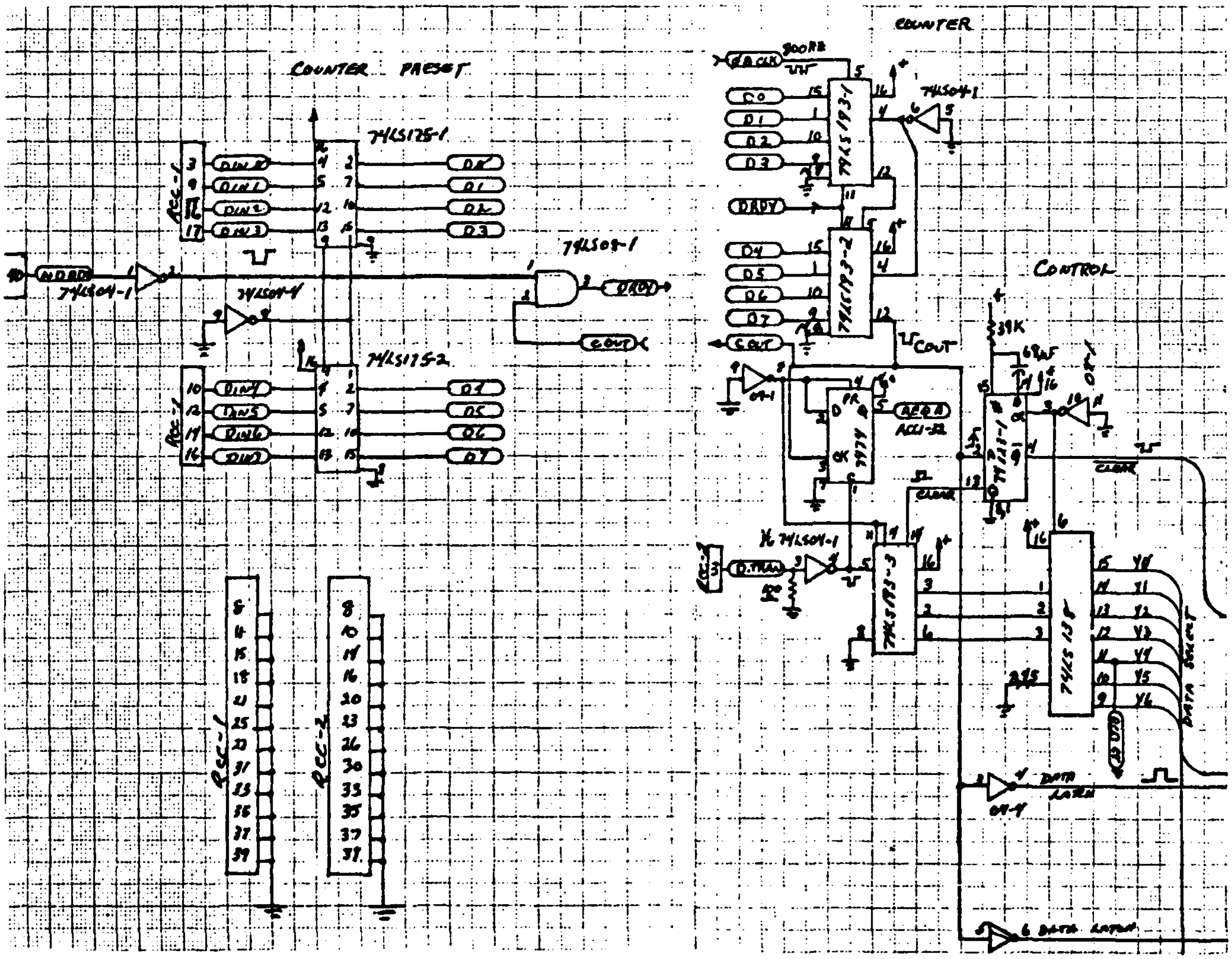


FIGURE 2.2 CONTROL/INTERRUPT CONTROLLER

H-478 puts out an 800 Hz B clock pulse that is used in determining the number of data cycles per second.

The interrupt controller consists of a counter preset circuit and an 8-bit count-up binary counter. The counter preset circuit uses two 74LS175 D-type flip-flops and a 74LS08 dual input AND gate. Since the information at the D input of the flip-flop can only be transferred to the Q outputs on the positive going edge of the load signal (NDRDY), the flip-flops act as a binary storage unit. With only two 74LS175's, the maximum number that can be stored is a binary 255. Both inputs to the AND gate, NDRDY and COUT, are normally high. The counter circuitry uses two 74LS193's cascaded to form an 8-bit counter that can be preset to a certain value. The 800 Hz clock is connected to the count-up input. The maximum number that can be counted is a binary 255.

To get, for example, four data cycles per second, the counter must be preset with a value so that with every 200 cycles of the B clock, the counter will produce a pulse that will shift data out of the interface. This means that the counter preset must be loaded, in this case, with a binary 56, the counter preset value. This value is shifted to the Q outputs of the 74LS175's on the positive going edge of the NDRDY pulse (DRDY is normally high, therefore the load signal is a negative going pulse) This negative going pulse causes the output of the AND gate, DRDY, to go low. The DRDY is connected to the load input of the counter. This loads or presets the counter to the preset value, the binary 56. With every 200 cycles (of the 800 Hz B clock) an output pulse is produced. This pulse, COUT, goes to the control section and also to the AND gate in the counter preset circuit. This drives the output, DRDY, low, which again loads the counter with the preset value. The computer only needs to load the preset value one time, or when it needs to be changed.

The control circuit consists of four chips; a 7474, a 74123, a 74LS138, and a 74LS193. The COUT pulse provides control for three functions in the control circuit and in the data storage/output section. The pulse is inverted to drive the enable of the octal D-type latch, in the output section, low. This latches the data in the storage section to the output section. At the same time, COUT is applied to the 74LS123 and to the 7474. The Q output of the 74LS123, a retriggerable mono-stable multivibrator, is inverted and applied to the clear input of the 74LS193s in the data storage section. This clears the counters after the data has been transferred into the output section. The Q output of the 7474, a D-type flip-flop, referred to as REQA, is used to signal the computer that data is ready to transmit. The computer responds with the DRTAN signal, which is a pulse train of seven bits. This is applied to the 74LS123. The output, which is a binary count, is connected to the 74LS138, a three to eight line decoder. This output, DATA SELECT, is applied to the output control on the 74LS373s, in the output section, in turn. This transfers the latched data to the data output bus. The DTRAN also resets the 7474. Another clear pulse, out of the 74LS123, clears the DTRAN counter. This insures that the counter will begin counting with zero at the start of each data cycle.

2.3 Data Storage/Output Section

The data storage/output section is shown in Figure 2-3. This section 1s made up of 74LSI93 and 74LS373 chips. The function of this section has already been explained except for the record count (RCD), which simply counts the number of data cycles transmitted. The count is incremented by the fifth data select pulse. Since the computer reads sixteen bits and only eight bits are provided by the record count circuit, a 74LS244 has been added to ensure that the last eight bits are zeros. The record count also serves as a fault indicator. If for some reason there is a malfunction of the data transmit signal, the record count will not increment.

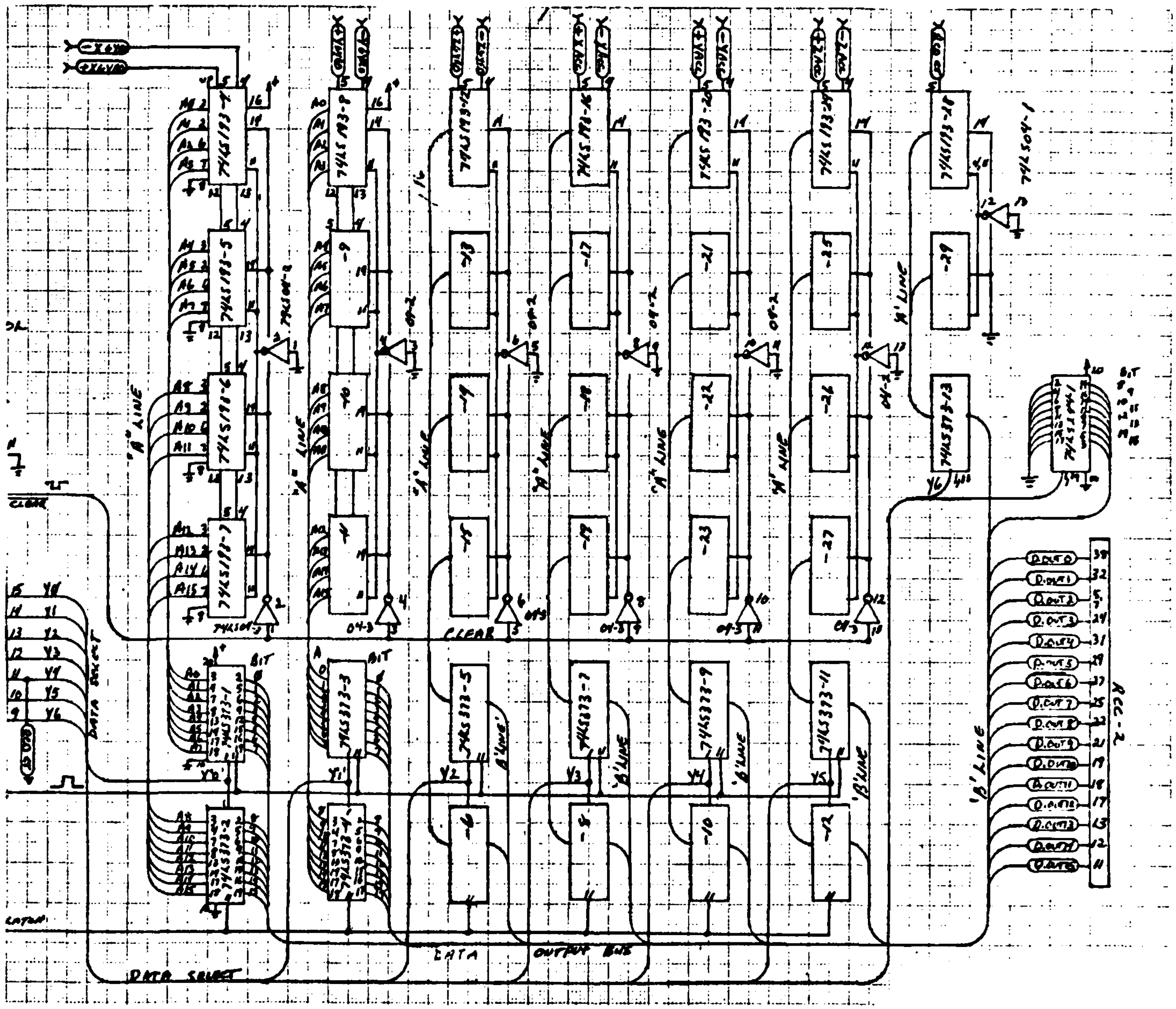


FIGURE 2.3 DATA STORAGE/OUTPUT SECTION

3.0 DATA PREPARATION

In order to facilitate the use of the ISA data output for alignment and navigation, it was necessary to (1) perform pre-averaging and (2) compute the sensor biases. These steps are described below.

3.1 Data Averaging

Figures 3-1 and 3-2 are functional schematic diagrams showing the numerical operation of the gyros and accelerometers. Both operate on the pendulum rebalance principle and produce pulse outputs proportional to the integral of angular velocity for the gyros and the integral of specific force for the accelerometers.

3.1.1 Gyros

If P_G is the number of gyro pulses counted in time T, i.e.

$$P_G = \sum n(t) \tag{3-1}$$

where n is the number of pulses per readout interval, then the average angular velocity referred to the input is

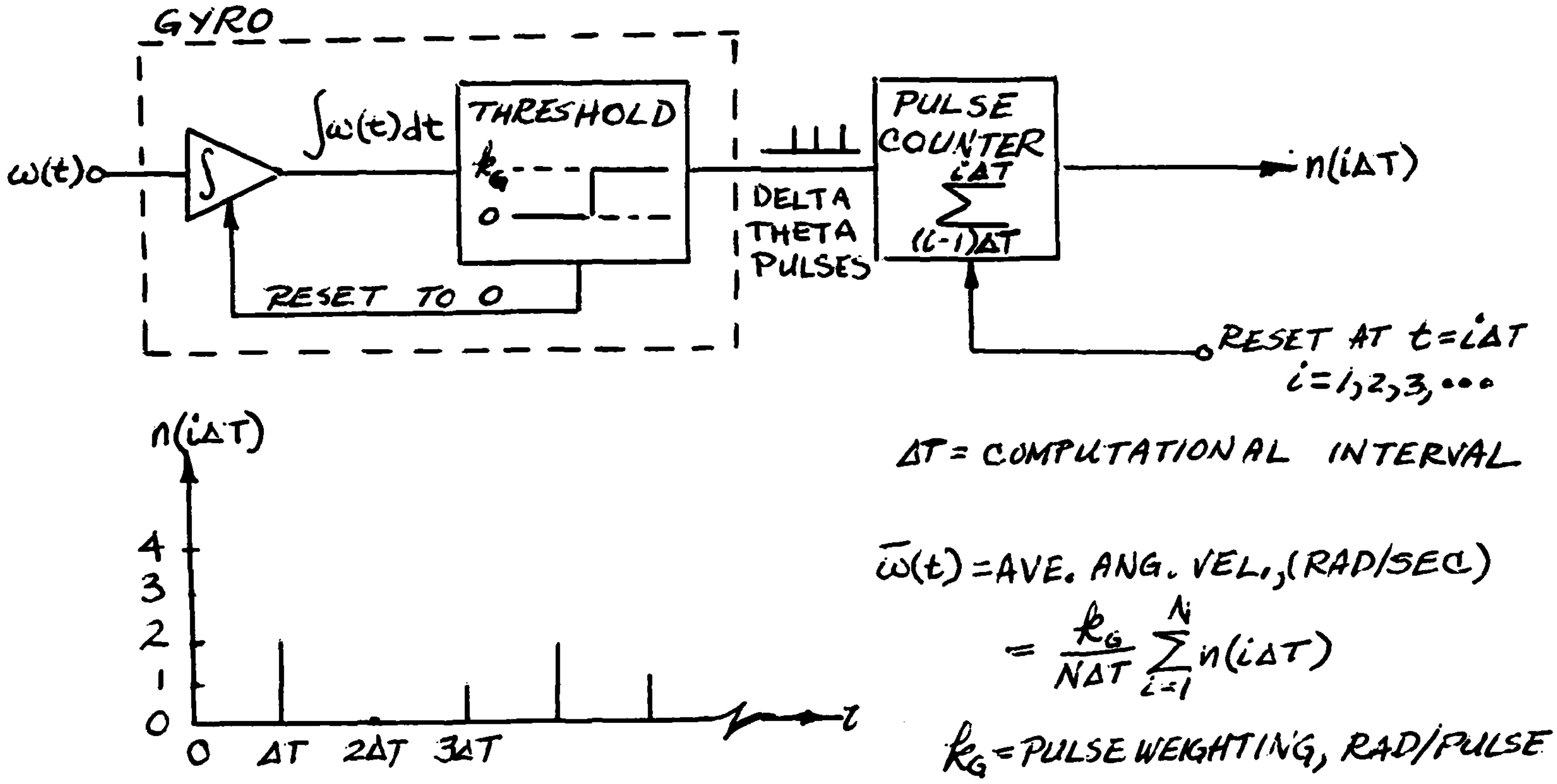


FIGURE 3-1 GYRO FUNCTIONAL SCHEMATIC

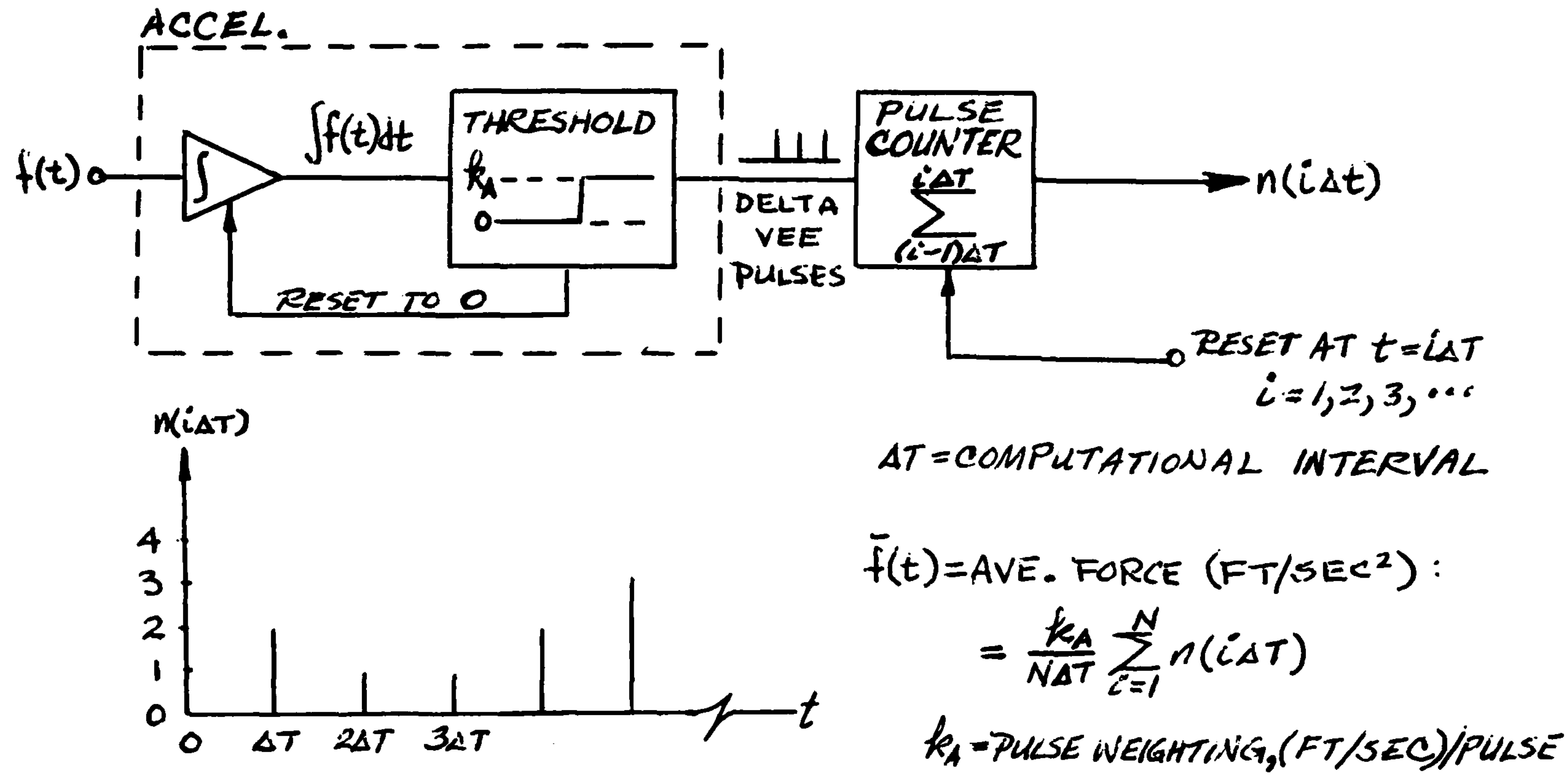


FIGURE 3-2 ACCELEROMETER FUNCTIONAL SCHEMATIC

$$\bar{\omega} = \frac{k_G}{T} P_G \tag{3-2}$$

where k_G is the pulse weighting.

The H-478 gyro channels, as obtained from Honeywell, had a pulse weighting of k_G = 62. sec/pulse. Considering, as we do for these experiments, earth rate to be the only angular velocity impingent upon the system, it is found that a pulse occurs roughly only once every 5 seconds on the north level axis. In order to compensate for this lack of sensitivity, the time interval T used in the above relationship was 5 minutes (300 seconds).

A more direct approach to increasing the sensitivity was that of reducing the pulse weighting factor k_G, thereby letting each pulse represent less angular rotation. This was accomplished by raising the amplifier gain, hence allowing the integral of w(t), Figure 3-2, to reach the fixed threshold level more frequently for a given w(t) input. For this purpose, the gain was multiplied by 10, giving k_G = 6.2 sec/pulse for certain of the runs described below.

3.1.2 Accelerometer

As can be seen by comparing Figures 3-1 and 3-2, from a functional viewpoint, the accelerometer and gyros are completely analogous. In this case, the average specific force referred to the input is

$$\bar{f} = \frac{k_A}{T} P_A$$

where P_A is the total pulse count in time T, i.e.

$$P_A = \sum n(t)$$

and k_A is the integral of specific force per pulse. For the H-478, k_A = .0625 ft/sec/pulse.

3.2 Bias Determination

In general, gyro and accelerometer biases were made by leveling the sensor triad and computing the difference between measured and computed outputs along or about the sensor forward, right wing and down (x, y, z) axes.

3.2.1 Gyro Bias

The north, east and down components of earth rate are

$$\begin{aligned} \omega_N &= \Omega \cos L \\ \omega_E &= 0 \\ \omega_D &= -\Omega \sin L \end{aligned} \qquad (3\text{-}3)$$

where is polar earth rate and L is geographic latitude.

The biases for the forward and right wing gyros are obtained with the given axis pointed north. This is done to minimize the effect of not knowing, precisely, the east or north directions. Thus if there is an azimuthal error , the sensed component of earth rate about the axis for which the bias is sought will be (see equation 3-3)

$$\tilde{\omega}_N = \Omega \cos L \cos \epsilon \qquad (3-4)$$

If, for example, $|\epsilon|$ is as large as 2°, the measured result will be diminished by only .061%. Hence a fair degree of uncertainty may be tolerated in pointing a given axis north for this purpose.

Because of the above, the x and y gyro axes are placed in the local level plane. The bias on the x (forward) gyro is determined by pointing the x-axis north and calculating

$$\delta \omega_x = \bar{\omega}_x - \Omega \cos L \qquad (3-5)$$

where the processed measurement ω_x is given by equation 3-2. The bias on the y-axis (right wing) gyro may be obtained by pointing it north giving

$$\delta w_y = \bar{w}_y - \Omega \cos L \qquad (3\text{-}6)$$

or pointing the left wing north (forward east) giving

$$\delta w_y = \bar{w}_y + \Omega \cos L \qquad (3\text{-}7)$$

The z-axis gyro bias (vertical and down) is given as

$$\delta w_z = \bar{w}_z + \Omega \cos L \qquad (3\text{-}8)$$

3.2.2 Accelerometer Bias

The north east and down components of gravity are

$$f_N = 0$$
$$f_E = 0$$
$$f_D = -g$$

Hence, with the x and y accelerometers level and z down, the biases are given by

$$\delta f_x = \bar{f}_x$$
$$\delta f_y = \bar{f}_y$$
$$\delta f_z = \bar{f}_z + 32.15479528$$

3.3 Data Averaging and Bias-Summary

A summary of the computations required for data preparation as discussed above is presented below.

3.3.1 Moving Averages

(a) ACCELEROMETER, x, y, z

$\bar{f} = \frac{k_A}{T} P_A$

where: k_A = .0625 (ft/sec)/pulse

P_A = number of accelerometer pulses in time T

$T \simeq 5$ min.

(b) GYRO, x, y, z

$w = \frac{k_G}{T} P_G$

where: k_G = .0003, .00003 rad/pulse

P_G = number of gyro pulses in time T

$T \simeq 5$ min.

3.3.2 Bias Determination

(a) COMPUTING X GYRO BIAS

x-axis is pointed north

$\delta w_x = \bar{w}_x - \Omega \cos L$

(b) COMPUTING ALL OTHER BIASES

x-axis pointed east

$\delta w_y = \bar{w}_y + \Omega \cos L$

$$\delta \omega_z = \bar{\omega}_z + \Omega \sin L$$

$$\delta f_x = \bar{f}_x$$

$$\delta f_y = \bar{f}_y$$

$$\delta f_z = \bar{f}_z + g$$

= .0000729115147 rad/sec

L = 39.783333 deg

g = 32.15479528 ft/sec^2

4.0 Alignment

The alignment of a strapdown inertial guidance assembly consists in determining the transformation from platform coordinates to local-level navigation coordinates. This is accomplished in two steps; (1) a coarse or analytic alignment, and (2) a fine alignment. These are described in the following paragraphs.

4.1 Coarse Alignment

Coarse alignment yields an initial value for the navigation to platform direction cosine matrix C_n^p, ignoring any effects from biases or noise. It is a purely analytic method which uses the gravity and earth rate vectors and their resultant outputs to deterministically solve for an approximation to C_n^p. Let f'_j and ω'_j (j = 1, 2, 3) represent the accelerometer and gyro outputs respectively. If we form the unit vectors

$$
\begin{aligned}
F'_j &= f'_j / |\underline{f}| \\
W'_j &= \omega'_j / |\underline{\omega}|
\end{aligned}
\qquad (4\text{-}1)
$$

the elements of the DCM are uniquely determined as

$$C'_{j1} = Z'_j / \cos L$$

$$C_{j2} = (W_j - F_j \sin L)/\cos L \tag{4-2}$$

$$C_{j3} = F_j$$

where

$$Z_1 = F_3 W_2 - F_2 W_3$$

$$Z_2 = F_1 W_3 - F_3 W_1 \tag{4-3}$$

$$Z_3 = F_2 W_1 - F_1 W_2$$

In the above, it is assumed that navigation coordinates are given as east-north-up. The platform coordinates may be designated arbitrarily. Appendix A gives a detailed derivation of the coarse alignment equations. These are the expressions used for coarse alignment with the H-478 sensor data.

4.2 Fine Alignment

The Kalman filter technique is employed to arrive at a final value for the navigation to platform direction cosine matrix C_n^p. The detailed development of the required equations is given in Appendix B. Since the sensor assembly is mounted on the Software Evaluation Laboratory (SEL) alignment table, which remains stationary with respect to the earth, the problem is considered to have no dynamics. Hence, only the measurement update expression for the filter is addressed. The filter states are taken to be the three angular errors $\delta\underline{\theta}_{np}$ in the estimate of the navigation to platform direction cosine matrix C_n^p. An updated estimate of C_n^p at time k is given in terms of its previous estimate at time k-1 as

$$\hat{C}_n^p(k) = \hat{C}_n^p(k-1)\left[I-(\delta\hat{\underline{\theta}}_{np}\times)+\frac{1}{2!}(\delta\hat{\underline{\theta}}_{np}\times)^2-\ldots\right] \quad (4\text{-}4)$$

where $\delta\underline{\theta}_{np}$ is the most recent estimate of the state vector. The current estimate of the state vector is given by

$$\delta\hat{\theta}_{np}(k) = \delta\hat{\underline{\theta}}_{np}(k-1) + K\underline{Z}_{RES} \quad (4\text{-}5)$$

where K is the matrix of optimum Kalman gains and $\underline{Z}_{RES}$ are the measurement residuals.

The three accelerometer and three gyro measurements $\underline{Z}$ are related to the imput specific force and angular velocity as

$$\underline{Z} \cong H\,\delta\underline{\theta}_{np} + \underline{v} \quad (4\text{-}6)$$

where $\underline{v}$ is a vector of white noises inherent in the sensors, and H is the 6x3 matrix given as

$$H = \begin{pmatrix} C_n^p\,(\underline{f}\times) \\ C_n^p\,(\underline{w}_{ip}\times) \end{pmatrix} \quad (4\text{-}7)$$

At any given time the estimated or predicted value of $\underline{Z}$ is given as

$$\hat{Z} = \begin{pmatrix} \hat{C}_n^{\beta} \, \underline{f}^n \\ \hat{C}_n^{p} \, \underline{\omega}_{i\beta}^n \end{pmatrix} \tag{4-8}$$

The measurement residual of equation (4-5) is thus given by

$$Z_{RES} = Z - \hat{Z} \tag{4-9}$$

The 3x3 Kalman gain matrix K is given at the kth instant of time as

$$K = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{4-10}$$

where P is the covariance matrix of the state vector estimates prior to the kth measurement update, and R is the 6x6 covariance matrix of gyro and accelerometer measurements. The covariance matrix of the state vector estimates after the kth measurement update is given by

$$P_k^+ = (I - KH_k) P_k^- \tag{4-11}$$

The filter was implemented using Carlson's triangular formulation. This method incorporates the measurements sequentially, thereby yielding an updated state vector and covariance matrix after each measurement.

5.0 NAVIGATION

The navigation software used with H-478 data was developed by the Draper Laboratory under contract to the Air Force Avionics Laboratory [1]. The Inertial Navigation Strapdown Simulator (INSS) program was developed as a tool for evaluating the effects of a tactical aircraft environment on the performance of the inertial components in a strapdown aircraft inertial measurement unit and hence, on the resulting INS navigation and attitude performance.

The INSS program was modified to accept H-478 data recorded on nine-track tape. Since this data was the output of gyro and accelerometer sensors, the portions of INSS which simulate inertial sensors were bypassed. The navigation algorithms use an alpha-wander coordinate system and third order altitude damping.

Denoting the wander azimuth coordinate system by the letter w, the velocity differential equation is

$$\dot{\underline{v}}^w = C^W_b \, \underline{f}^b + \underline{g}^W - (\underline{\omega}^w_{ew} + 2\underline{\omega}^W_{ie})^* \, \underline{v}^W - C_2 \Delta h \underline{u}_h - a_h \underline{u}_h \qquad (5\text{-}1)$$

where

$\Delta h = h - h_B$

h_B = barometric altitude

$\dot{a}$ = $C_3 \Delta h$

C_2 = 2nd order altitude damping constant

C_3 = 3rd order altitude damping constant

$\underline{u}_h$ = (0 0 1) = unit vertical vector

$\underline{\omega}_{ew}$ = angular rate vector of wander azimuth wrt earth coordinates

$\underline{\omega}_{ie}$ = angular rate vector of earth wrt inertial coordinates

$\underline{g}$ = gravity vector

$\underline{f}^p$ = force vector from the accelerometers

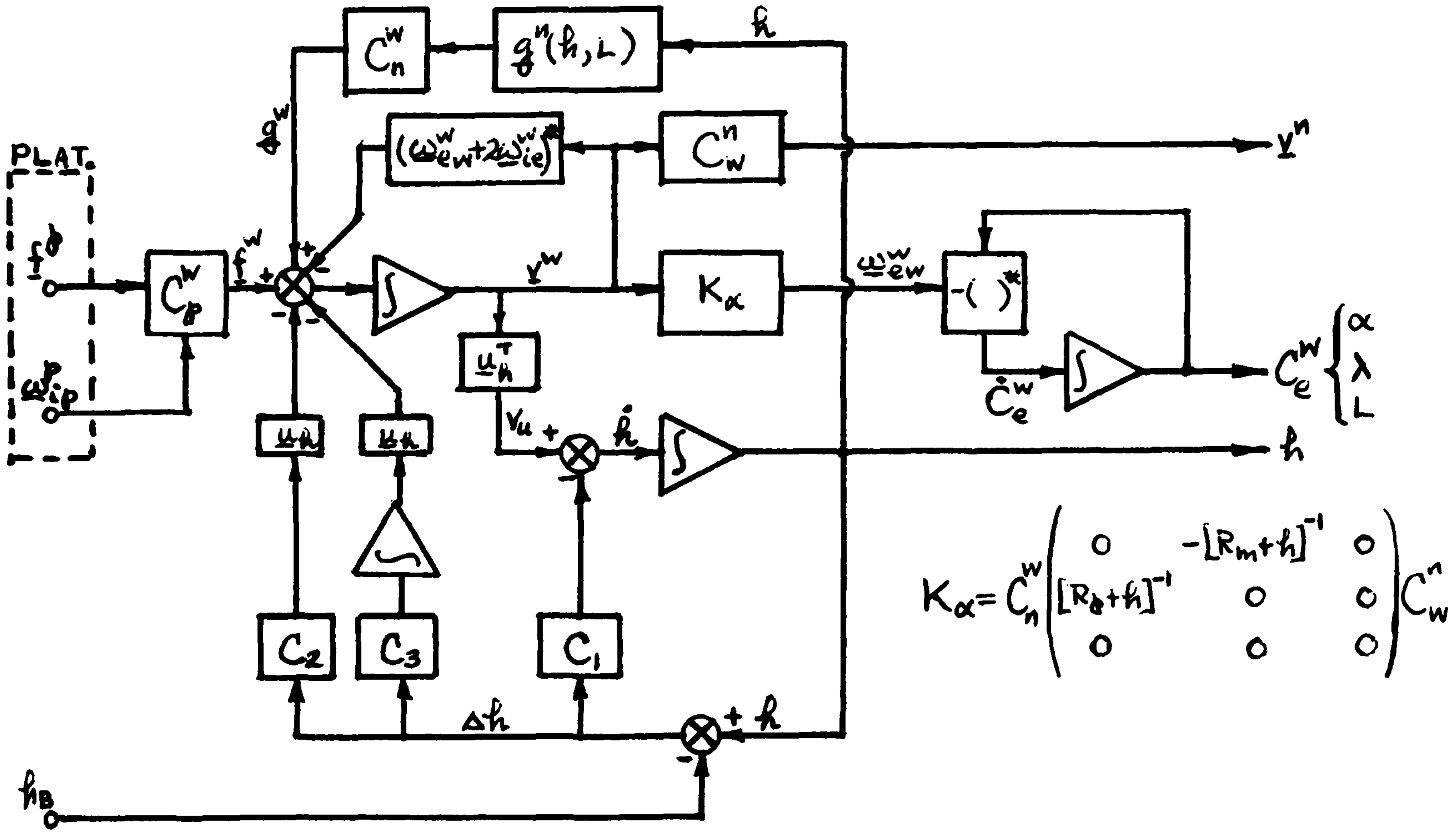
C_p^w = direction cosine matrix transformation from platform (or body) to wander azimuth coordinates

$(\underline{x})^*$ = the skew-symmetric form of a vector $\underline{x}$.

Figure 5-1 is a mathematical flow diagram showing the navigation computations. The initial value of the transformation of C_p^w is found by the alignment procedure previously discussed. This transformation is seen to be a function of the gyro output vector $\underline{\omega}_{ip}^p$.

The differential equation which propagates the horizontal position is

$$\dot{C}_e^w = -(\underline{\omega}_{ew}^w)^* C_e^w \qquad (5\text{-}2)$$



$$K_\alpha = C^w_n \begin{pmatrix} 0 & -[R_m+h]^{-1} & 0 \\ [R_p+h]^{-1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} C^n_w$$

FIGURE 5-1 INSS NAVIGATION MATHEMATICAL FLOW DIAGRAM where

$\underline{\omega}_{ew}$ = the angular velocity vector of the wander azimuth wrt earth coordinates (craft rate)

C_e^w = the direction cosine transformation from earth to wander azimuth coordinates

This matrix is of the form, Figure 5-2,

$$C_e^w = \begin{pmatrix} \cos L\cos(\lambda-\lambda_0) & \cos L\sin(\lambda-\lambda_0) & \sin L \\ C_{21} & C_{22} & \cos L\sin\alpha \\ C_{31} & C_{32} & \cos L\cos\alpha \end{pmatrix} \quad (5\text{-}3)$$

where

L = geographic latitude

λ_0 = initial longitude

λ = current longitude

α = wander angle

Horizontal position is extracted from the C_e^w matrix, equation 5-3, as

$$L = \arctan\left(\frac{C_{13}}{\sqrt{C_{11}^2+C_{12}^2}}\right)$$

$$\lambda = \arctan\left(\frac{C_{12}}{C_{11}}\right) + \lambda_0 \quad (5\text{-}4)$$

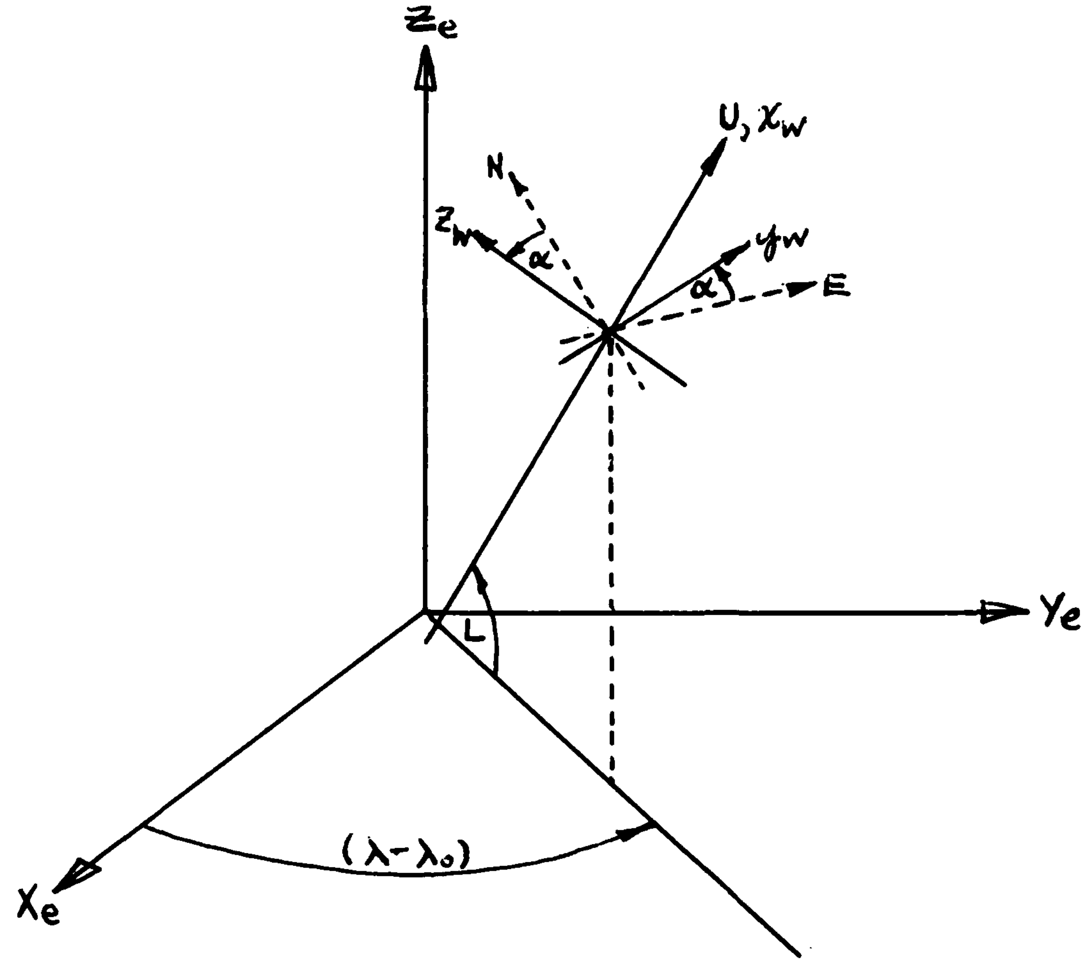


FIGURE 5-2 RELATIONSHIP BETWEEN EARTH AND WANDER AZIMUTH COORDINATE SYSTEMS

Vertical position, i.e. altitude, is given by the differential equation

$$\dot{h} = v_{h} - C_{1}\Delta h \qquad (5\text{-}5)$$

where

v_{h} = the vertical component of the velocity vector $\underline{v}$, equation 5-1

C_{1} = the 1st order altitude damping constant

Also obtainable from C_{e}^{w} is the wander angle

$$\alpha = \arctan\left(\frac{C_{23}}{C_{33}}\right) \qquad (5\text{-}6)$$

This is required to determine attitude as is shown below.

The attitude at any instant may be extracted from the transformation C_{p}^{w} of equation 5-1. denoting the platform frame as Foreward-Leftwing-Up and the navigation frame as Up-East-North (when α=0),

$$C_{p}^{w} = \begin{pmatrix} \sin P & \sin R\cos P & \cos R\cos P \\ \cos P\sin(H+\alpha) & C_{22} & C_{23} \\ \cos P\cos(H+\alpha) & C_{32} & C_{33} \end{pmatrix} \qquad (5\text{-}7)$$

where R = roll

P = pitch

H = heading wrt north

Therefore it is seen that the attitude is expressible as

$$
\begin{aligned}
R &= \arctan\left(\frac{C_{12}}{C_{13}}\right) \\
P &= \arctan\left(C_{11} \Big/ \sqrt{C_{12}^2 + C_{13}^2} \right) \qquad (5\text{-}8) \\
H &= \arctan\left(\frac{C_{21}}{C_{31}}\right) - \alpha
\end{aligned}
$$

6.0 EXPERIMENTAL RESULTS

In order to exercise the H-478 and the associated software described above, the sensor assembly was nominally oriented as shown in Figure 6-1. The experimental problem, using only the gyro and accelerometer outputs, is to (1) determine the orientation of the ISA and (2) determine position, velocity and attitude over a period of time using this stationary trajectory.

In each of two cases the three accelerometer and three gyro pulse counts were recorded onto nine-track high density magnetic tape. At the outset, for each case, the x or forward axis was pointed to the north in order to obtain the x-gyro bias as explained in 3.2.1. Approximately one hour is spent in that position. The x-axis is then turned eastward where it remains for the rest of the recording period. In this position, data was taken for approximately one hour on run number one and approximately two hours on run number two From this data, i.e. with x eastward, we obtain (1) bias quantities for the y and z gyros and all accelerometers, (2) estimates of each sensor's noise variance, (3) alignment and (4) navigation.

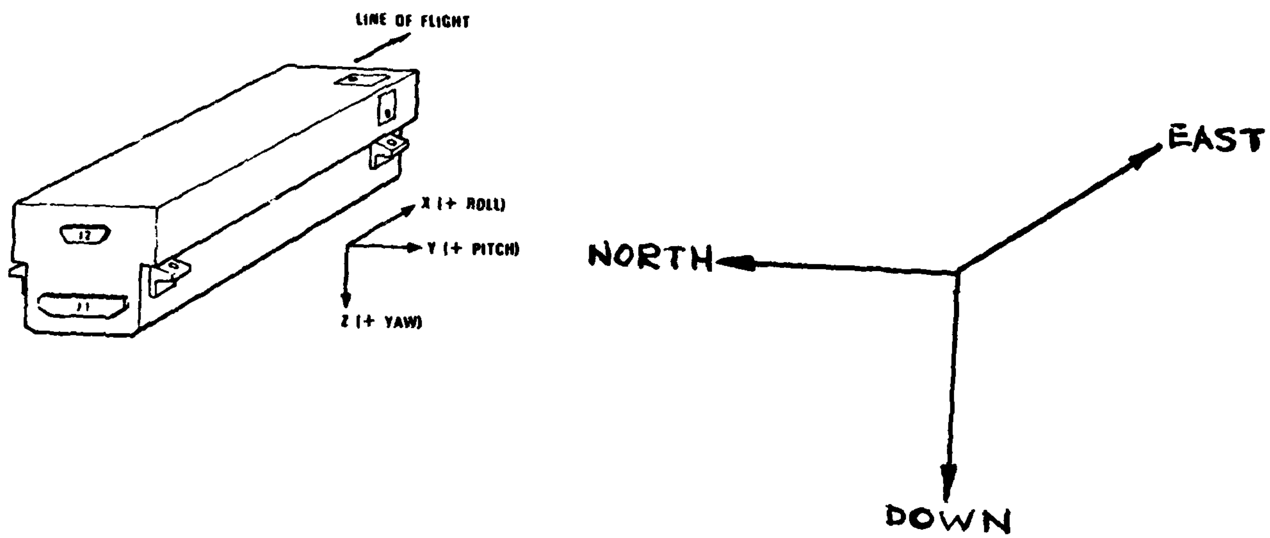


FIGURE 6-1 H-478 INERTIAL SENSOR ASSEMBLY ORIENTATION

6.1 Run # 1

This run, recorded on tape X00540, is with the gyro scale factors unaltered; 62. arcsec/pulse. As in all cases, the pulse data is averaged over 5 minutes. The data interval is .25 seconds. Consequently, 1200 data points are averaged to obtain each gyro and accelerometer reading. The biases and noise standard deviations are given in Table 6-1 below.

AXIS	GYRO (RAD/SEC)		ACCEL (FT/SEC^2)	
	BIAS	NOISE DEV	BIAS	NOISE DEV
X	2.10X10	2.0X15	1.04X10	1.0X10
Y	-2.58X10	1.67X10	6.48X10	6.0X10
Z	2.17X10	6.0X10	3.33X10	1.05X10

Table 6-1 Measurements for X00540

Figures 6-2 through 6-4 show the fine alignment process as a function of time in terms of heading, pitch and roll as determined from the navigation to platform coordinate transformation.

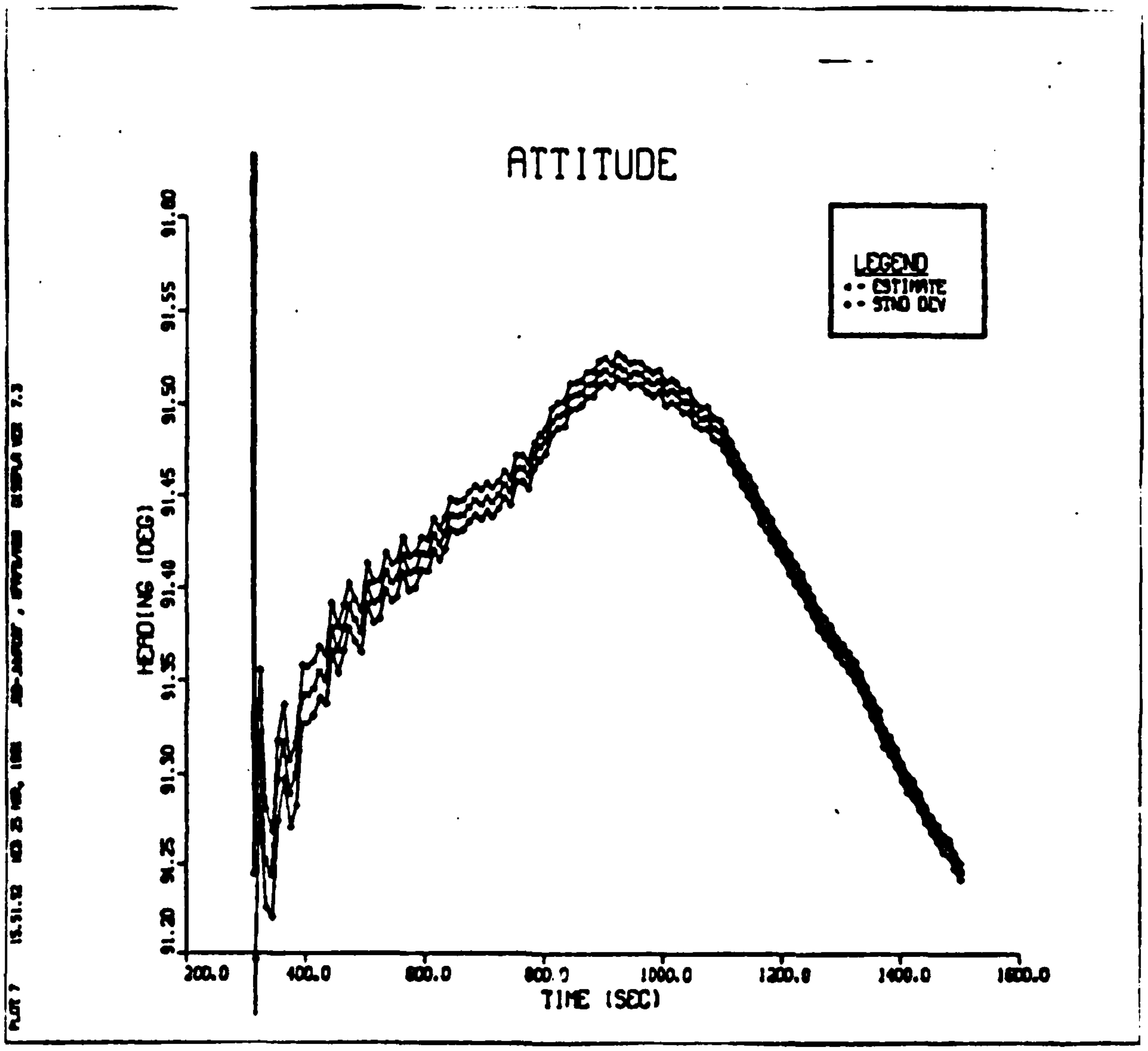


FIGURE 6-2 ALIGNMENT HEADING VS TIME

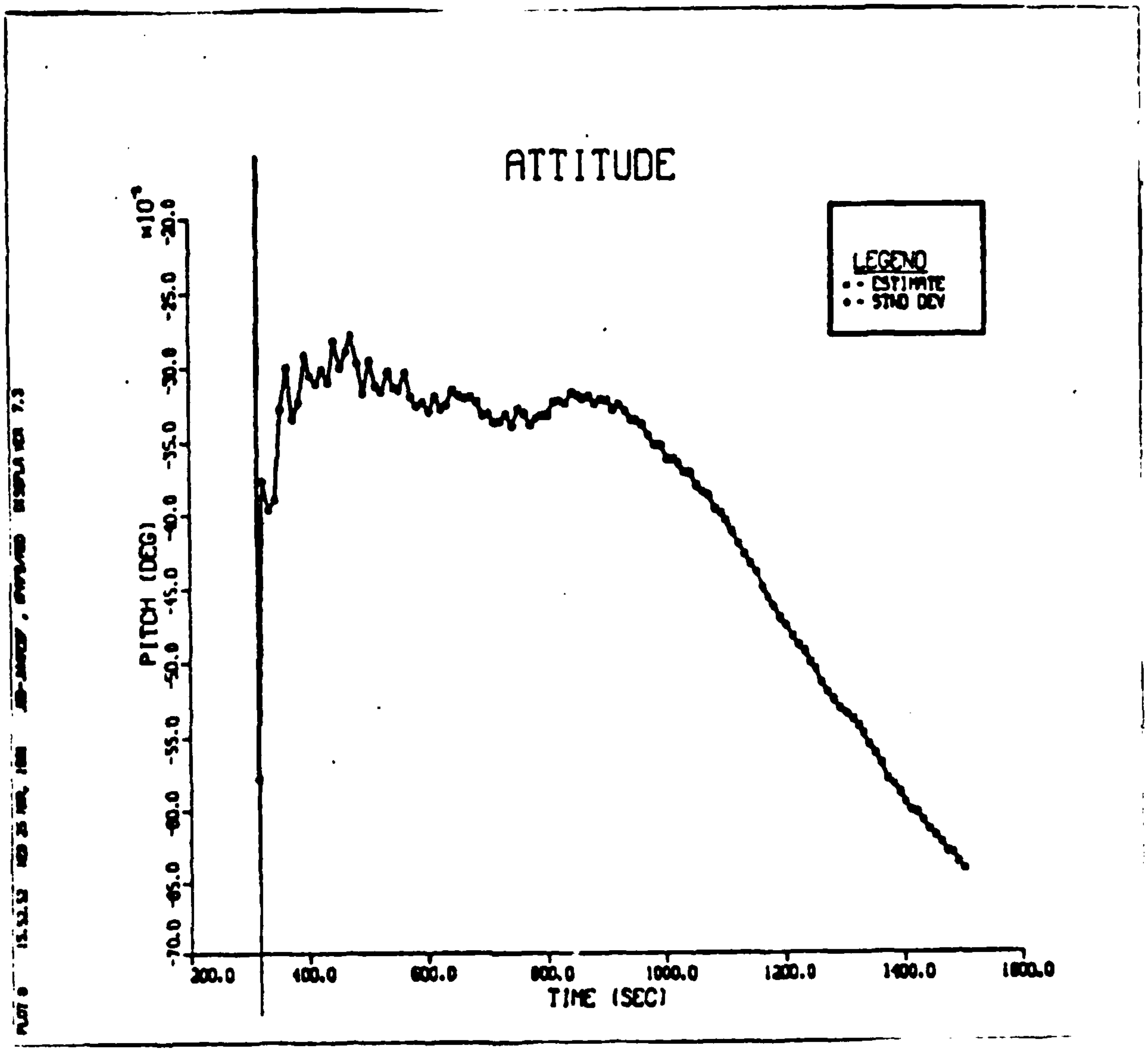


FIGURE 6-3 ALIGNMENT PITCH VS TIME

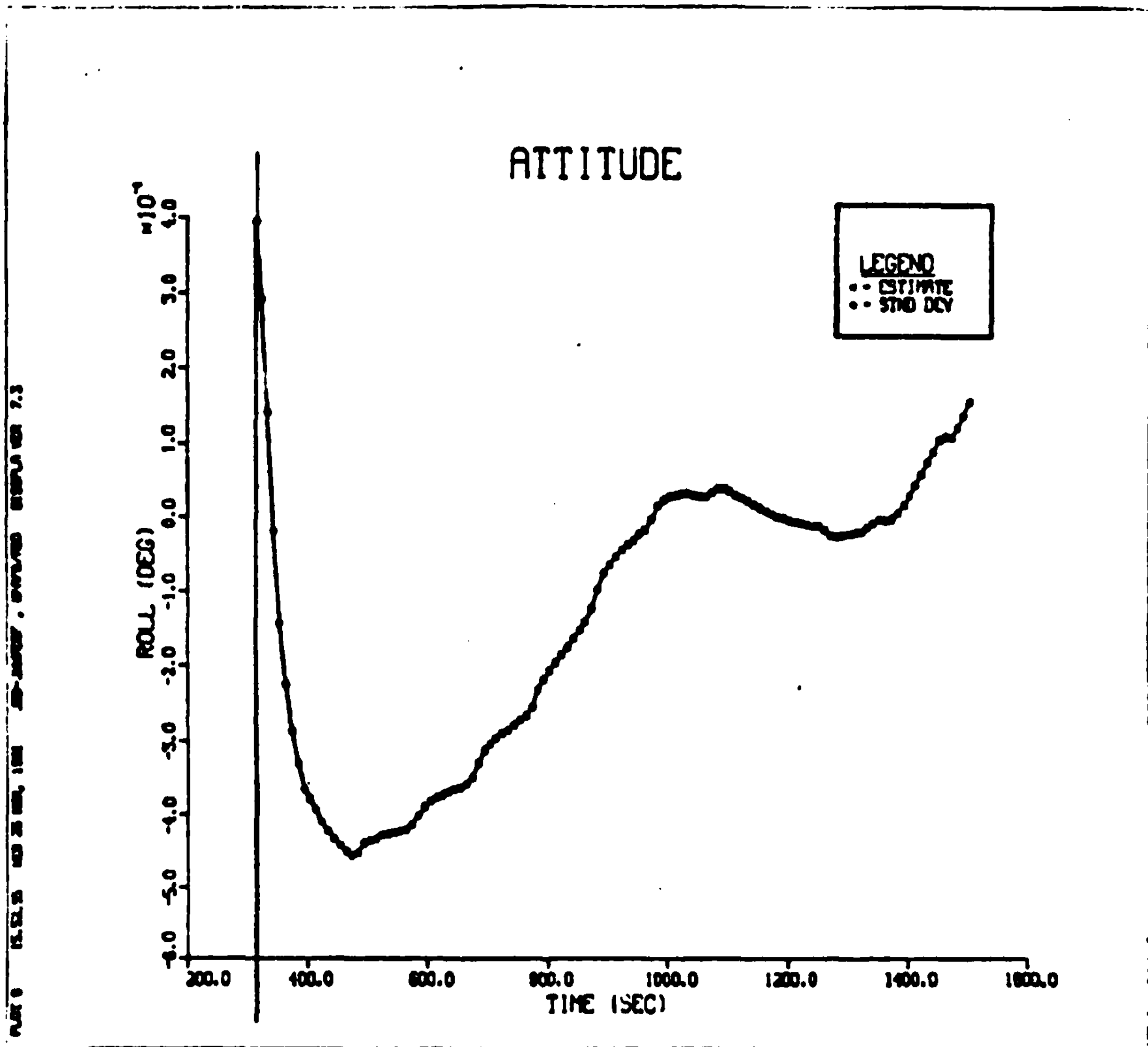


FIGURE 6-4 ALIGNMENT-ROLL VS TIME

It will be noted that the pitch and roll variations, Figures 6-3 and 6-4, are very small, less than one thousandth of a degree. Also, the uncertainty associated with roll and pitch is negligible.

The variations in heading versus time are more pronounced than for roll or pitch, showing about a 2°/hr z-gyro drift from 1100 to 1500 seconds.

The attitude used for alignment in the navigation mode was

Heading	91.25
Pitch	-.000065
Roll	.0002

Figures 6-5 through 6-8 show the errors in navigation. The latitude and longitude plots give approximately a 4 n.m./hour position error. The velocity errors grow to about 15 ft/sec for east velocity and 25 ft/sec for north velocity.

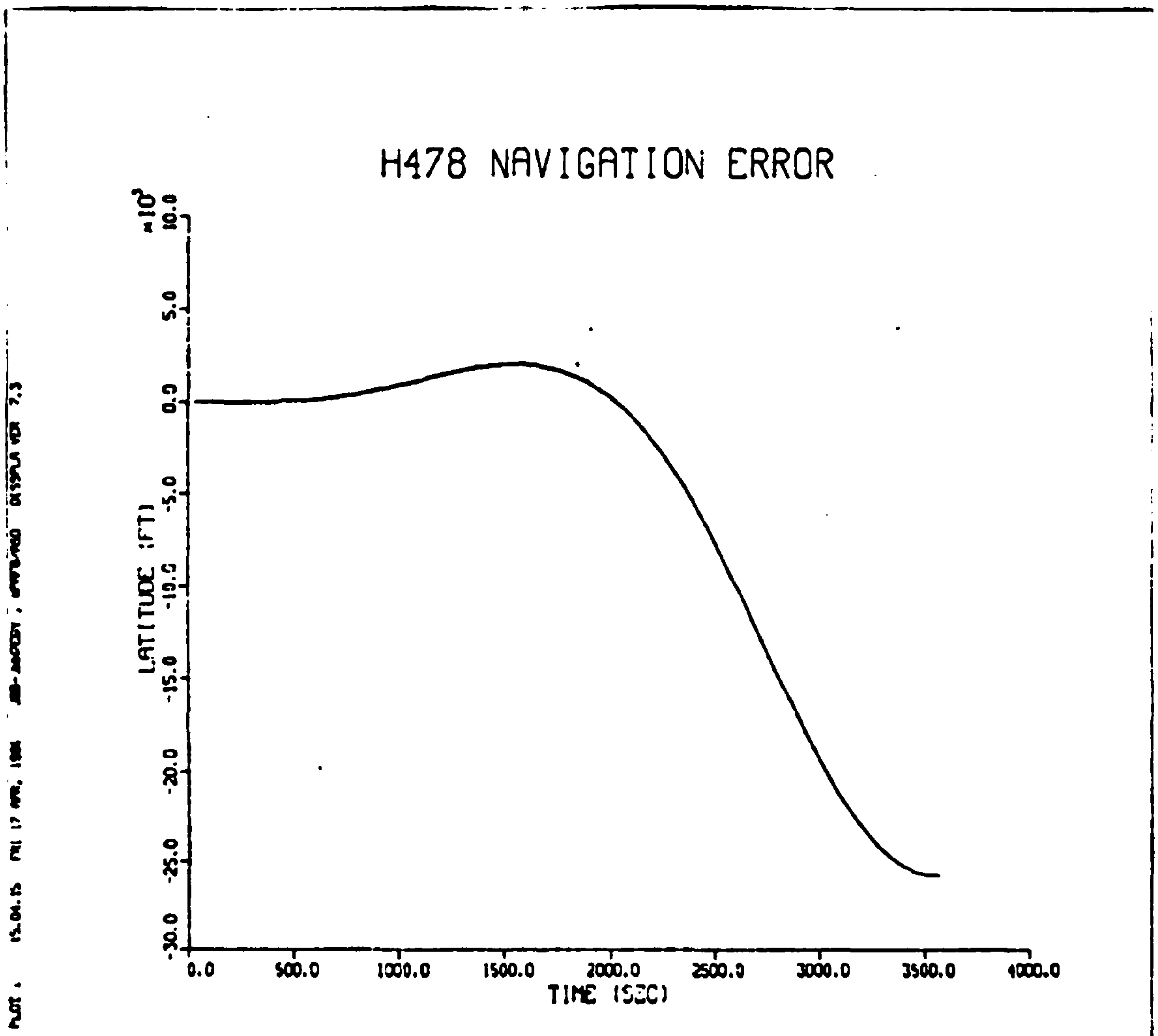


FIGURE 6-5 NAVIGATION-LATITUDE VS TIME

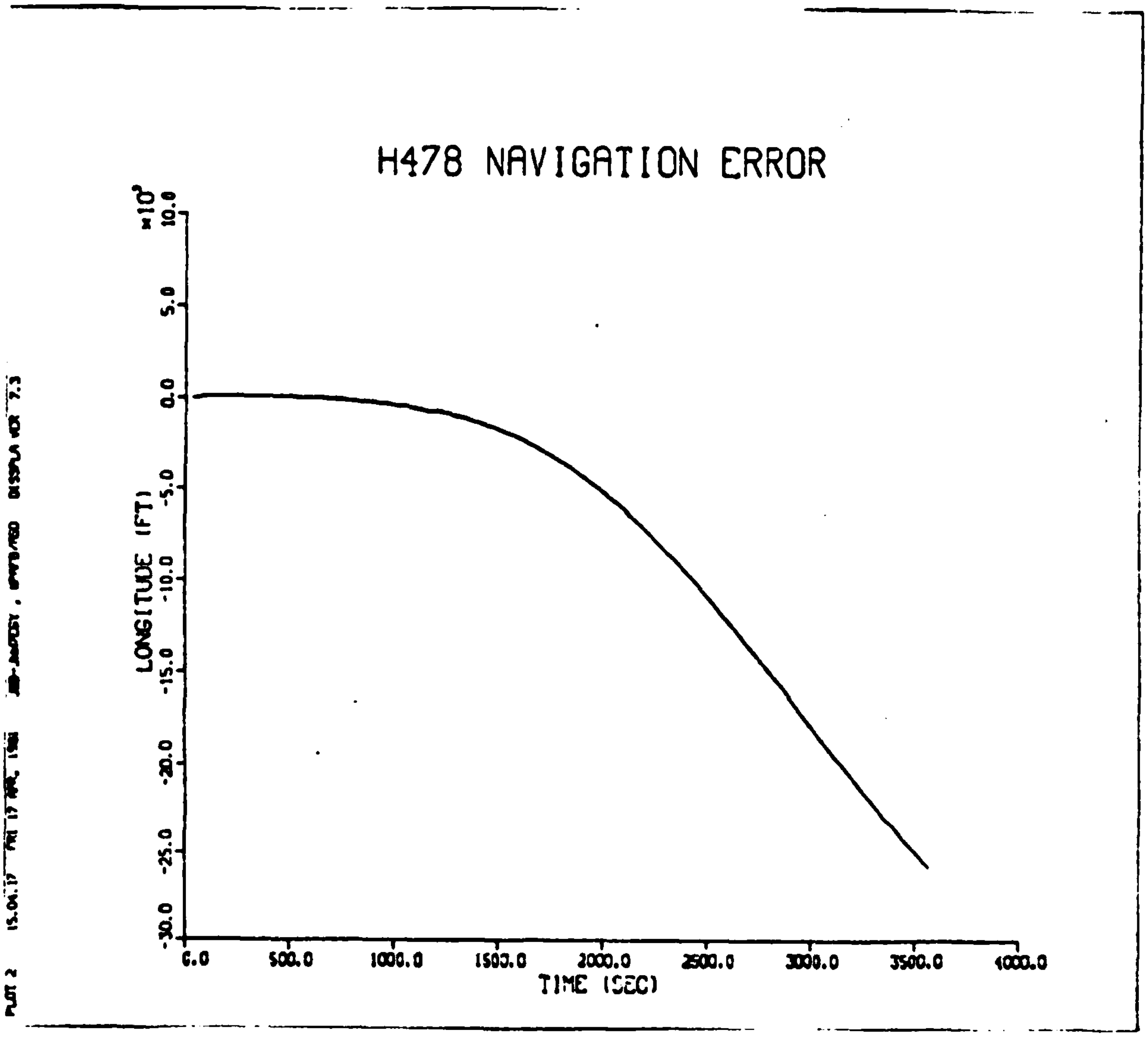


FIGURE 6.6 NAVIGATION-LONGITUDE VS TIME

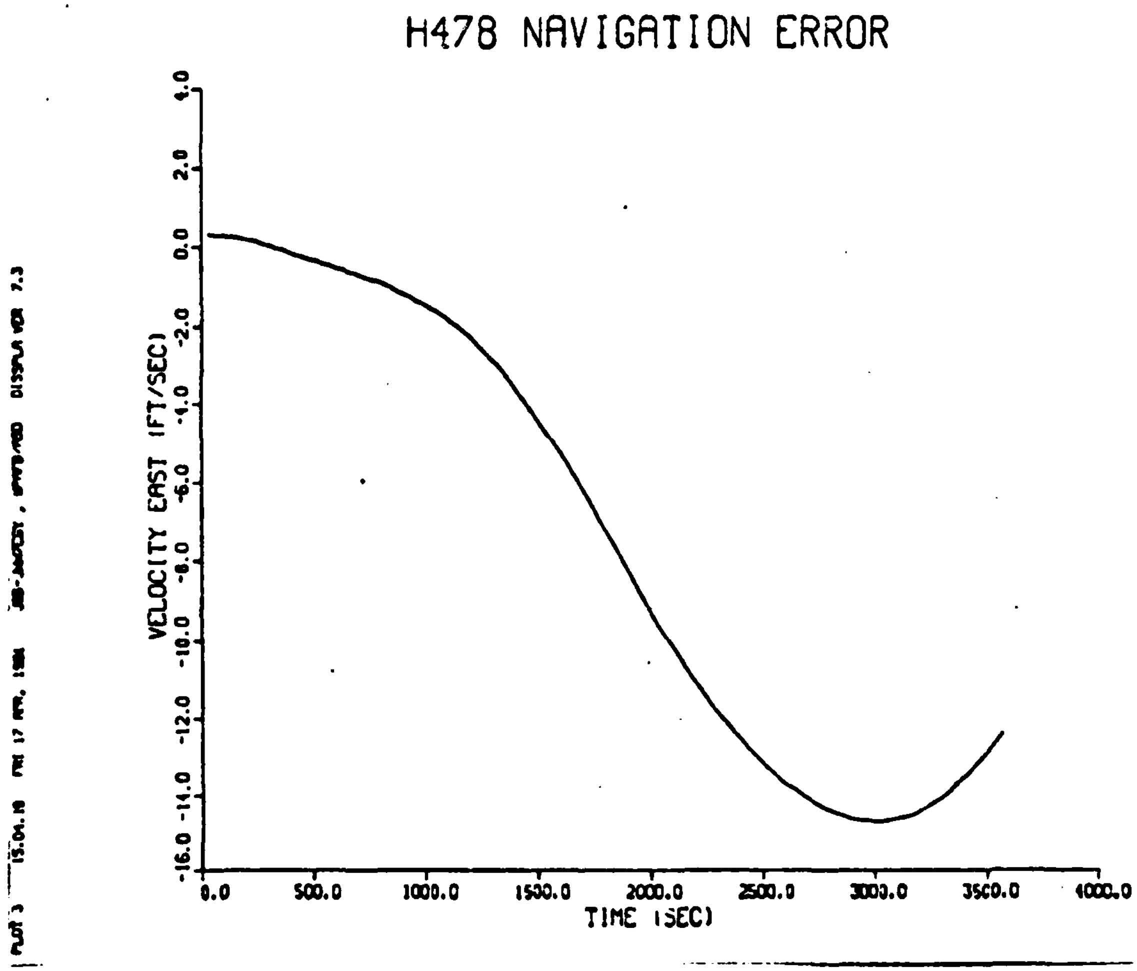


FIGURE 6-7 NAVIGATION-VELOCITY EAST VS TIME

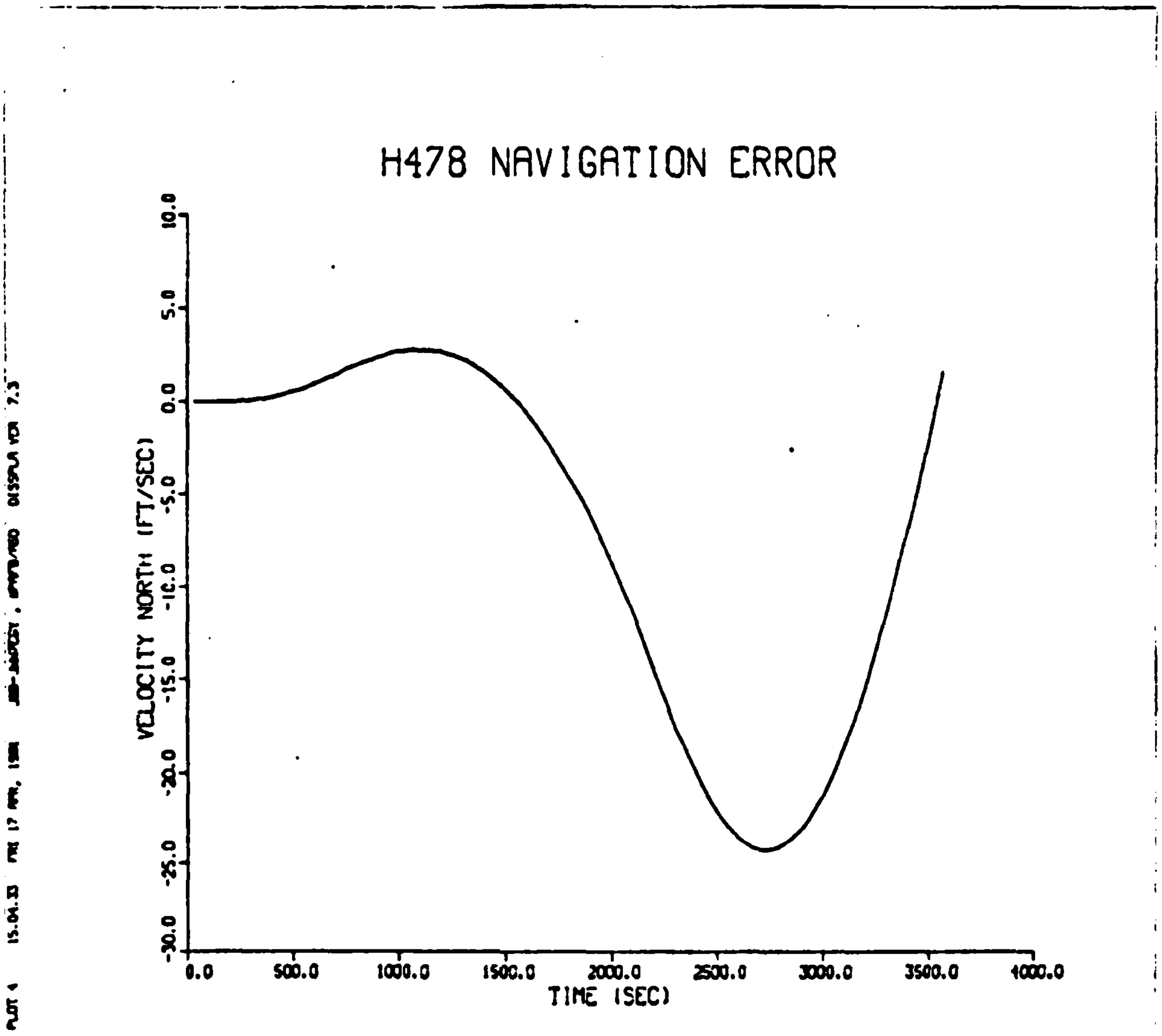


FIGURE 6.8 NAVIGATION-VELOCITY NORTH VS TIME

6.2 Run # 2

For this run, the gyro scale factor was changed to 6.2 arc sec/pulse. Also, the recording was lengthened from one to two hours for the navigation run. Data averaging was the same as in Run # 1. Table 6-2 gives the measured biases and noise standard deviations obtained from this run.

AXIS	GYRO (RAD/SEC)		ACCEL (FT/SEC)	
	BIAS	NOISE DEV	BIAS	NOISE DEV
X	8.75X10	3.5X15	1.0X10	1.0X10
Y	2.58X10	1.0X10	-2.75X10	1.0X10
Z	-2.10X10	3.5X10	0.0	5.0X10

Table 6-2 Measurements for X00097

The variation in heading, Figure 6-9, indicates a z-gyro drift of about 1^{o} /hr. As in Run # 1, the alignment shows negligible pitch and roll, Figures 6-10 and 6-11.

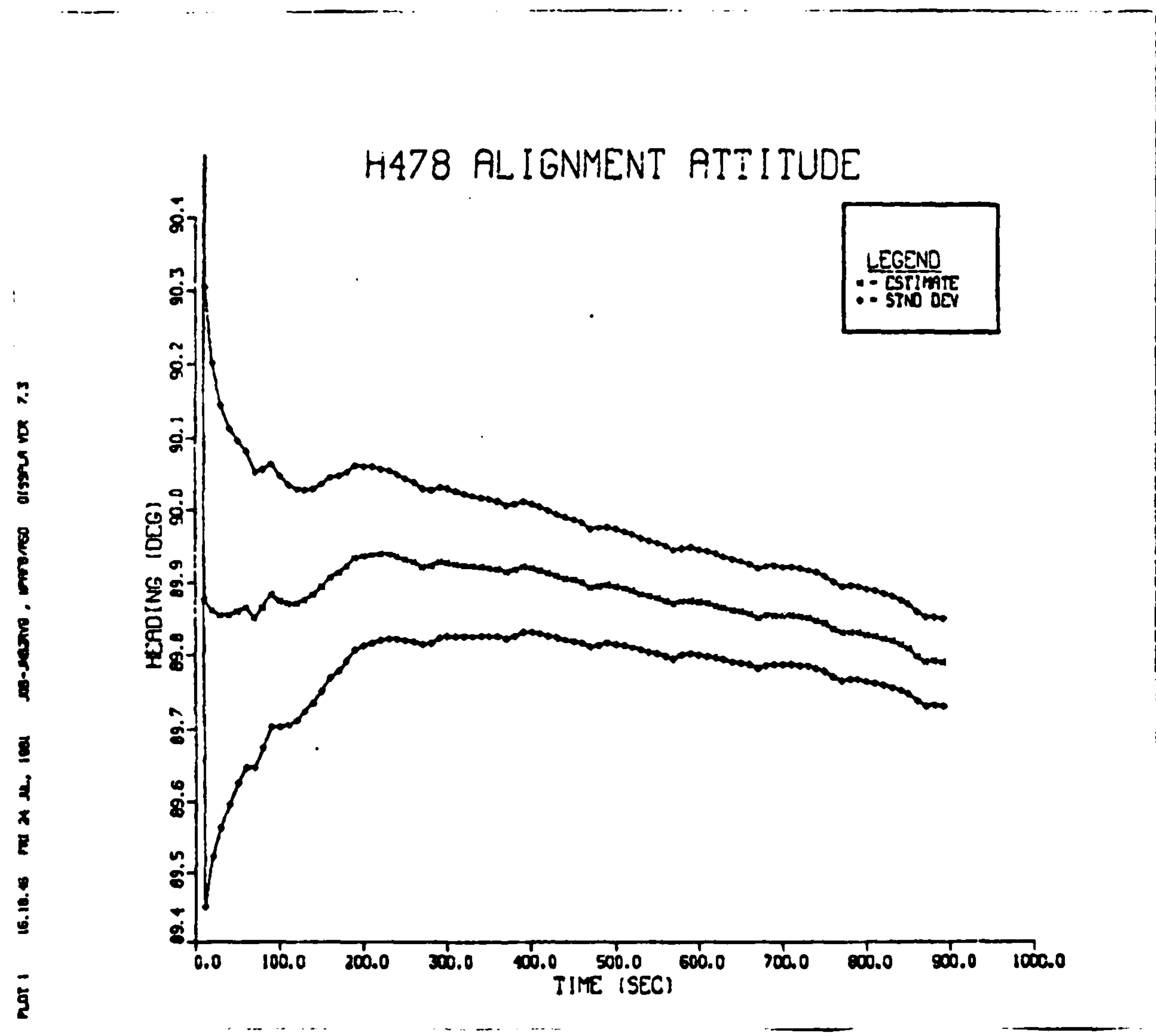


FIGURE 6-9 ALIGNMENT-HEADING VS TIME

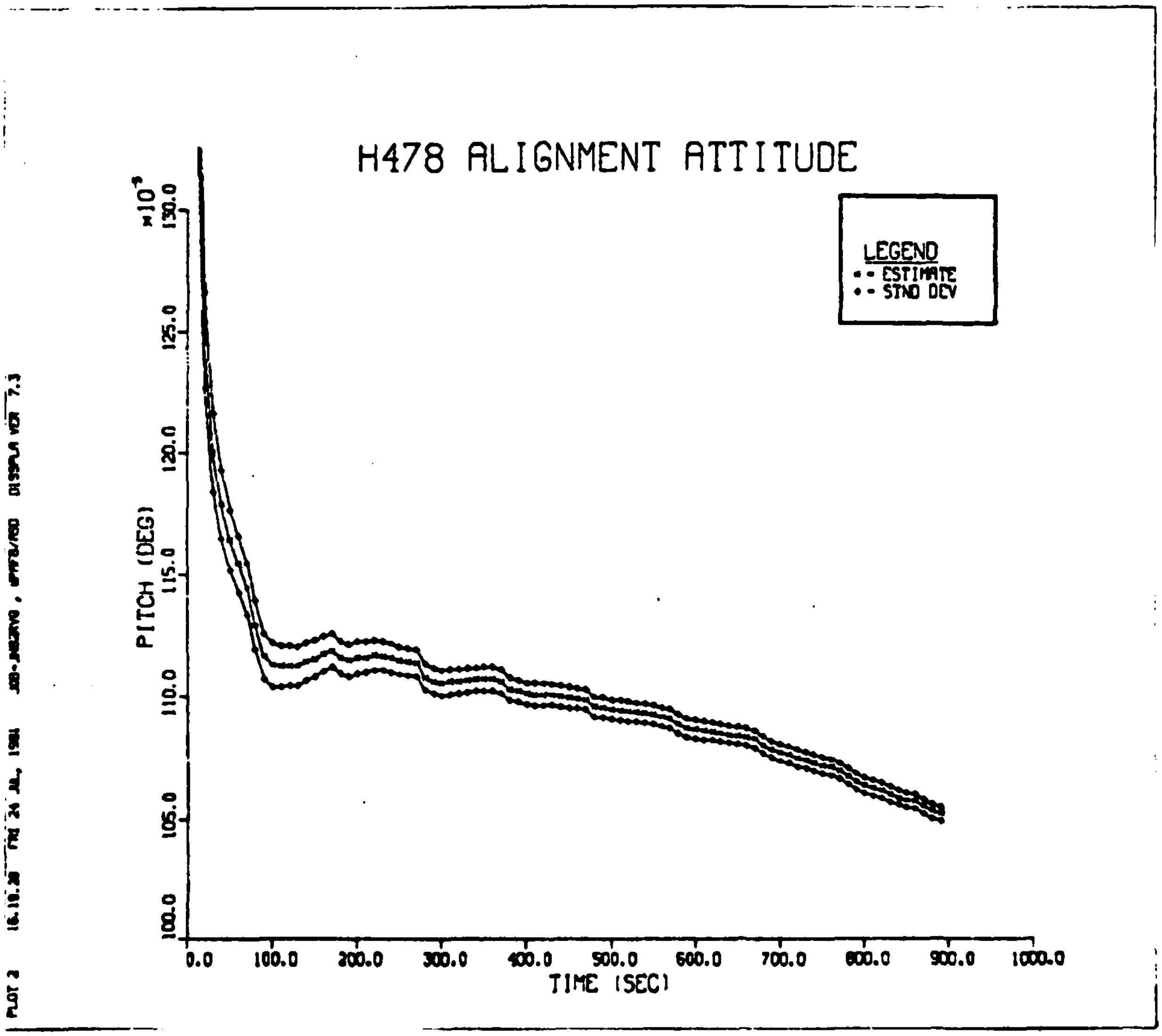


FIGURE 6-10 ALIGNMENT-PITCH VS TIME

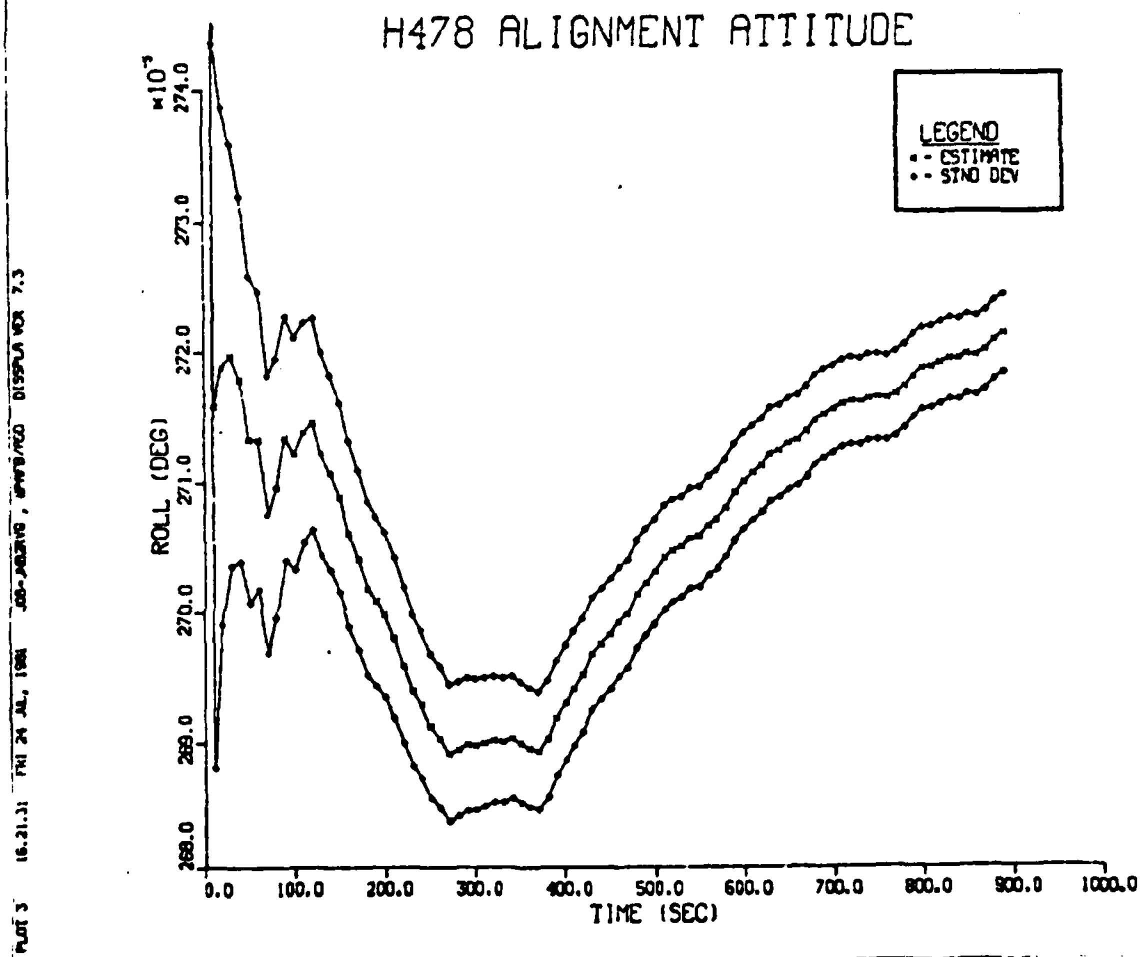


FIGURE 6.11 ALIGNMENT-ROLL VS TIME

The attitude used for alignment in the navigation mode was

Heading	89.8
Pitch	.001
Roll	.0027

Note that there is approximately a 1.5 difference in heading between the two runs.

The Schuler oscillation (84 minutes = 5040 seconds) can be seen in the navigation error plots, Figures 6-12 through 6-15. These are typical of the plots obtained when an initial attitude error exists. The latitude error is 50. n.m./hr. The maximum velocity errors are 18. ft/sec and 150. ft/sec for the east and north directions respectively.

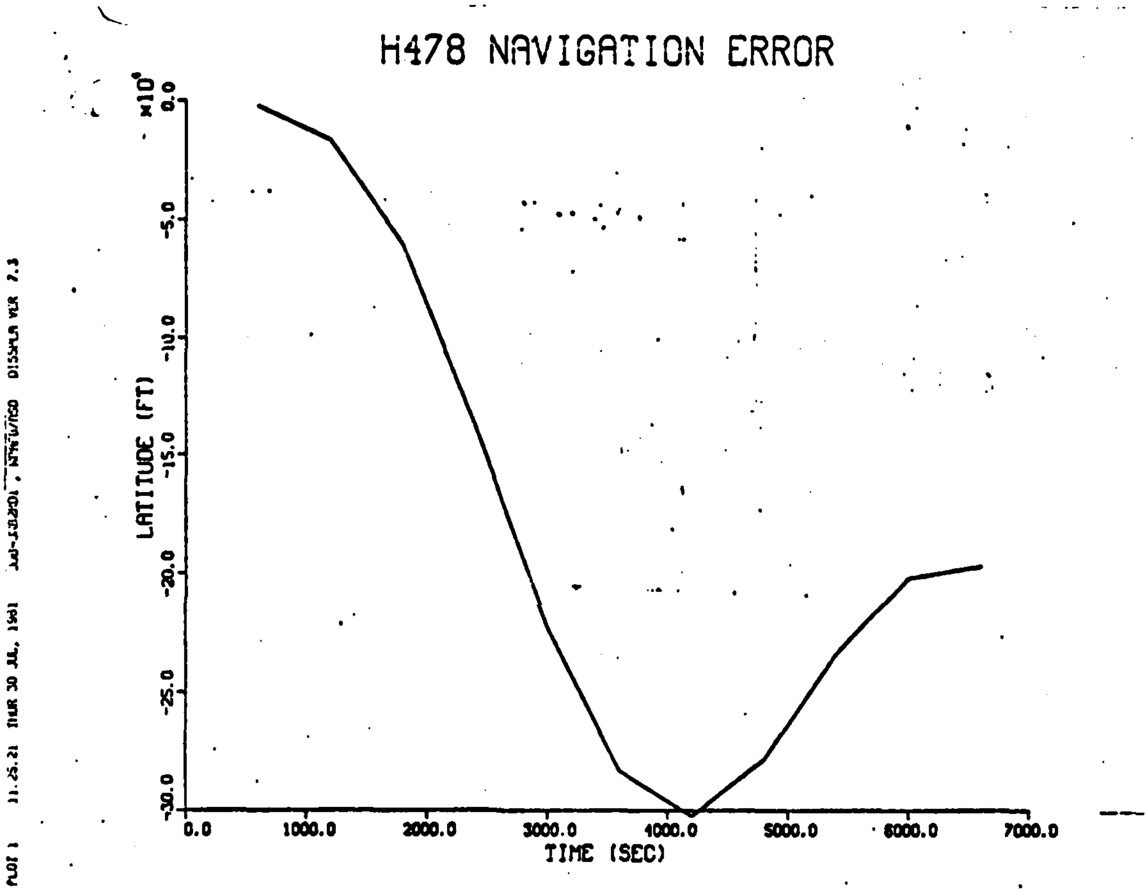


FIGURE 6.12 NAVIGATION-LATITUDE VS TIME

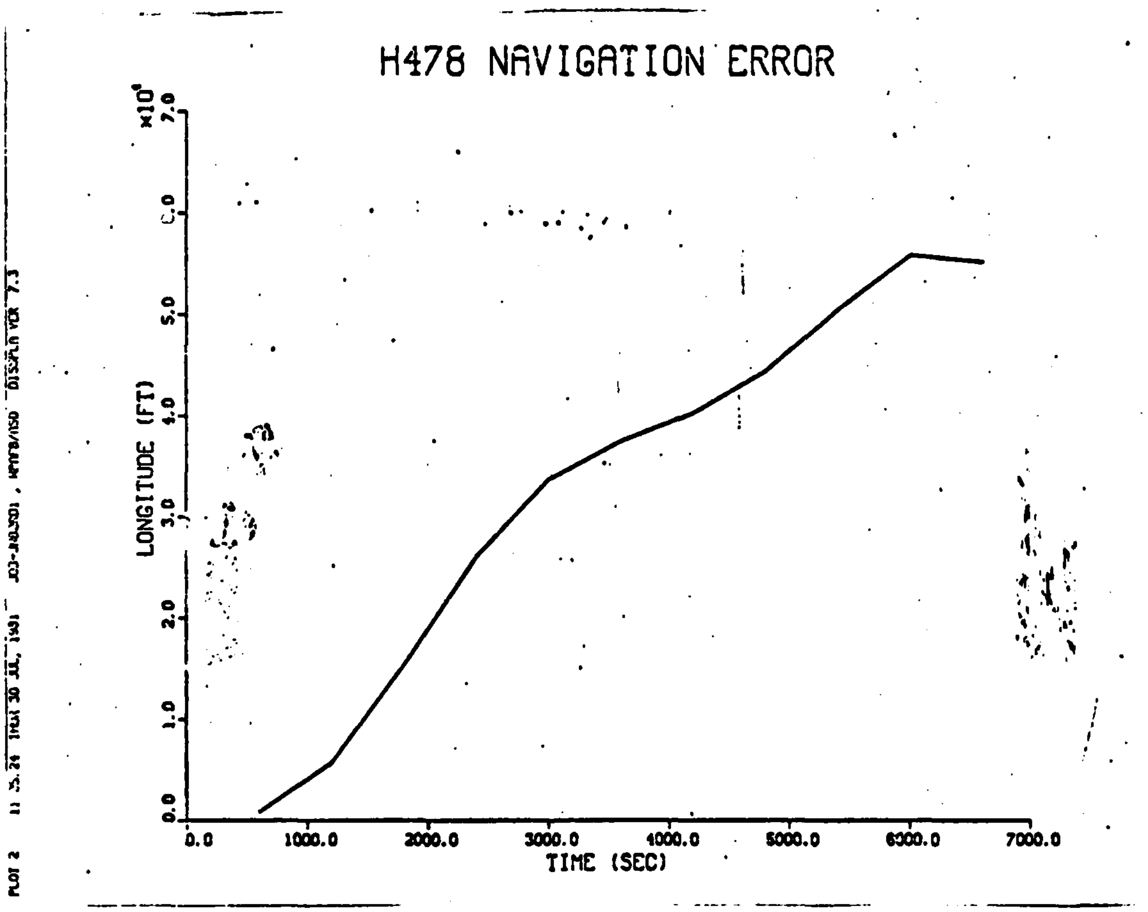


FIGURE 6-13 NAVIGATION-LONGITUDE VS TIME

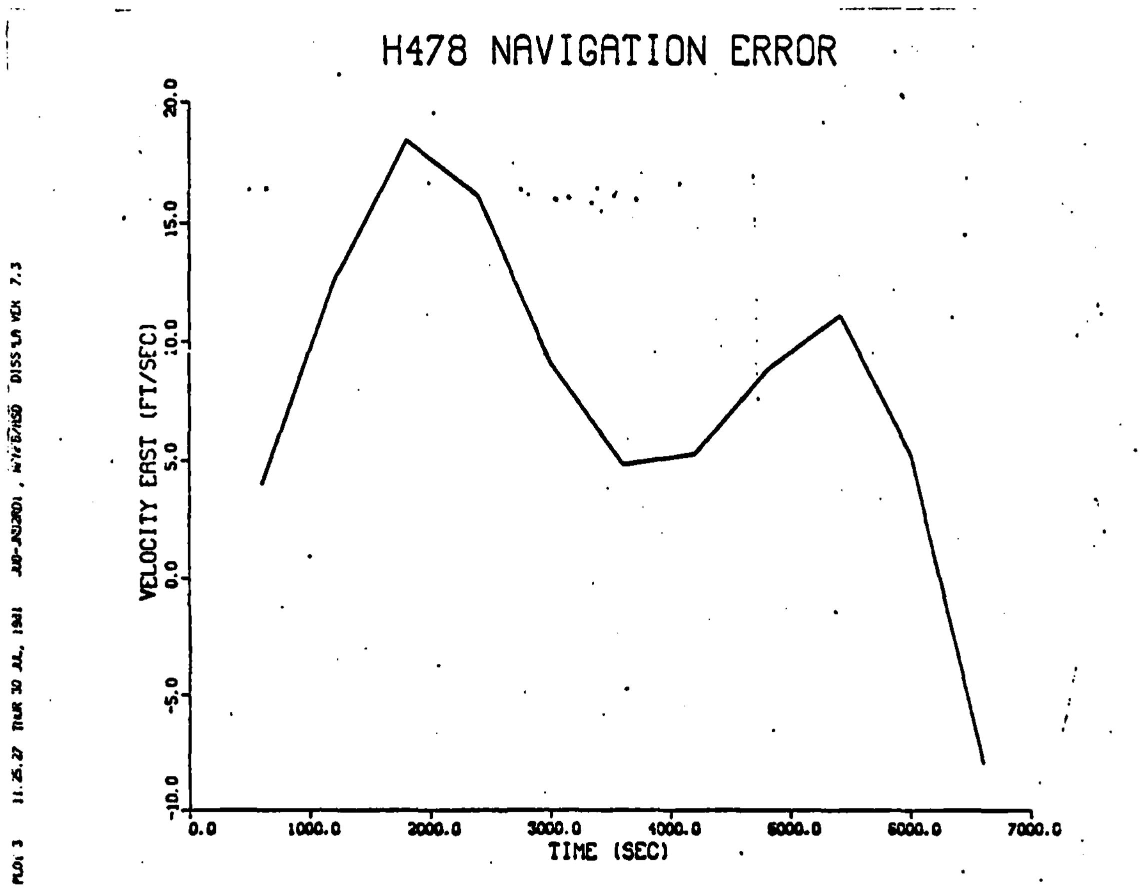


FIGURE 6.14 NAVIGATION VELOCITY EAST VS TIME

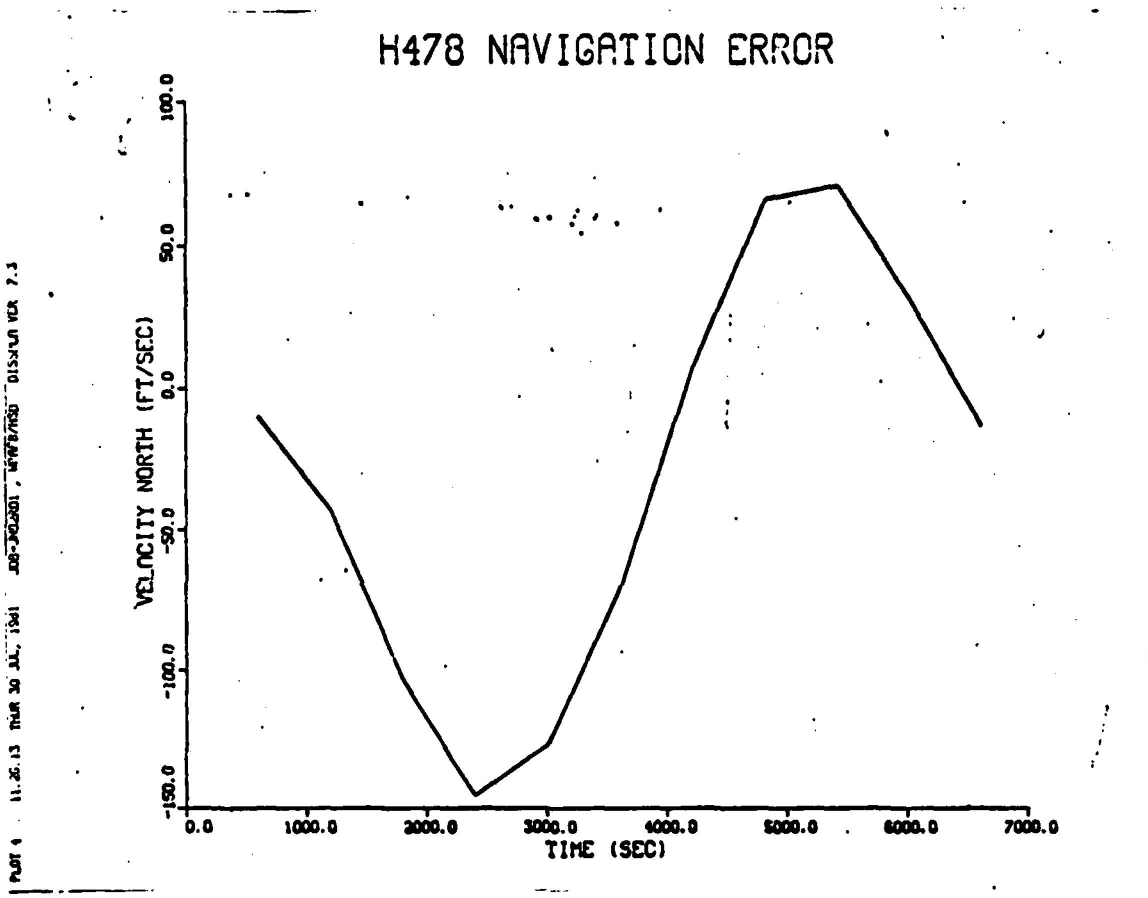


FIGURE 6-15 NAVIGATION-VELOCITY NORTH VS TIME

APPENDIX A

Equations For Strapdown Coarse Alignment

APPENDIX A

Equations For Strapdown Coarse Alignment

The direction cosine matrix relating navigation to platform coordinates, C_n^p , is formed by using the relationships

$$
\begin{aligned}
\underline{f}^p &= C_n^p\ \underline{f}^n \\
\underline{w}^p &= C_n^p\ \underline{w}^n \qquad\qquad (1) \\
\underline{z}^p &= C_n^p(\underline{f}^n x \underline{w}^n)
\end{aligned}
$$

where

$\underline{f}$ is the specific force vector and

$\underline{w}$ is the angular velocity vector of the platform wrt inertial space.

In equations (1), $\underline{f}^n$ and $\underline{w}^n$ are inputs to the accelerometers and gyros respectively, while $\underline{f}^p$ and $\underline{w}^p$ are the corresponding outputs. Equations (1) can be expressed in one equation as

$$
\begin{pmatrix} f_j^p \\ w_j^p \\ z_j^p \end{pmatrix} = \begin{pmatrix} f_1^n & f_2^n & f_3^n \\ w_1^n & w_2^n & w_3^n \\ (\underline{f}^n x \underline{w}^n)_1 & (\underline{f}^n x \underline{w}^n)_2 & (\underline{f}^n x \underline{w}^n)_3 \end{pmatrix} \begin{pmatrix} C_{j1} \\ C_{j2} \\ C_{j3} \end{pmatrix} \qquad (2)
$$

where $j = 1, 2, 3$ and C_{jk} is the jkth element of the DCM, C_n^p. Equation (2) may be solved explicitly for the DCM elements, thus

$$\begin{pmatrix} C_{j1} \\ C_{j2} \\ C_{j3} \end{pmatrix} = \begin{pmatrix} f_1^n & f_2^n & f_3^n \\ \omega_1^n & \omega_2^n & \omega_3^n \\ (\underline{f} \times \underline{\omega})_1 & (\underline{f} \times \underline{\omega})_2 & (\underline{f} \times \underline{\omega})_3 \end{pmatrix}^{-1} \begin{pmatrix} f_j^p \\ \omega_j^p \\ z_j^p \end{pmatrix} \tag{3}$$

Now the input vetors, $\underline{f}$ and $\underline{\omega}$ in east-north-up navigation coordinates and for a stationary system are given as

$$\begin{aligned} \underline{f}^n &= (\quad 0 \qquad 0 \qquad g\ )^T \\ \underline{\omega}^n &= (\quad 0 \quad \Omega\cos L \quad \Omega\sin L)^T \end{aligned} \tag{4}$$

where g is gravity, Ω is earth rate and L is geographic latitude. As a result of (4), the vector $\underline{f}^n \times \underline{\omega}^n$ is given by

$$\underline{f}^n \times \underline{\omega}^n = (-g\Omega\cos L \quad 0 \quad 0)^T \tag{5}$$

Using (4) and (5) in (3), we have

$$\begin{pmatrix} C_{j1} \\ C_{j2} \\ C_{j3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & g \\ 0 & \Omega\cos L & \Omega\sin L \\ -g\cos L & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} f_j^p \\ \omega_j^p \\ z_j^p \end{pmatrix} \tag{6}$$

Inverting the matrix in (6) gives

$$\begin{pmatrix} C_{j1} \\ C_{j2} \\ C_{j3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dfrac{-1}{g\Omega\cos L} \\ \dfrac{-\sin L}{g\cos L} & \dfrac{1}{\Omega\cos L} & 0 \\ 1/g & 0 & 0 \end{pmatrix} \begin{pmatrix} f_j^p \\ \omega_j^p \\ z_j^p \end{pmatrix} \tag{7}$$

We note at this point that

$$\begin{aligned} \underline{z}^p &= C_n^p\, \underline{f}^n \times \underline{\omega}^n = \underline{f}^p \times \omega^p \\ &= (f_2^p\omega_3^p - f_3^p\omega_2^p \quad f_3^p\omega_1^p - f_1^p\omega_3^p \quad f_1^p\omega_2^p - f_2^p\omega_1^p)^T \end{aligned} \tag{8}$$

Now multiplying out equation (7) yields

$$\begin{aligned} C_{j1} &= \frac{-z_j^p}{g\cos L} \\ C_{j2} &= \left(\frac{\omega_j^p}{\Omega} - \frac{f_j^p \sin L}{g}\right) \Big/ \cos L \\ C_{j3} &= \frac{f_j^p}{g} \end{aligned} \tag{9}$$

where from (8) z_j is calculated using the instrument outputs as

$$\begin{aligned} z_1^p &= f_2^p\ \omega_3^p - f_3^p\ \omega_2^p \\ z_2^p &= f_3^p\ \omega_1^p - f_1^p\ \omega_3^p \\ z_3^p &= f_1^p\ \omega_2^p - f_2^p\ \omega_1^p \end{aligned} \tag{10}$$

We can effect a simplification in (9) by noting that

$$|\underline{f}| = g$$
$$|\underline{w}| = \Omega \qquad (11)$$

Hence equations (9) can be expressed in terms of unit vectors. Thus if we define unit vectors in the platform frame

$$F_j^p \triangleq f_j / |\underline{f}|$$
$$W_j^p \triangleq \omega_j / |\underline{\omega}| \qquad (12)$$

then

$$C_{j1} = -Z_j^p / \cos L$$
$$C_{j2} = (W_j^p - F_j^p \sin L) / \cos L \qquad (13)$$
$$C_{j3} = F_j^p$$

where

$$Z_1^p = F_2^p W_3^p - F_3^p W_2^p$$
$$Z_2^p = F_3^p W_1^p - F_1^p W_3^p \qquad (14)$$
$$Z_3^p = F_1^p W_2^p - F_2^p W_1^p$$

We see that it is unnecessary to supply explicit numerical values for g and Ω. Writing out the matrix explicitly, we have in terms of the unit vectors, (equation (12)),

$$C_n^{\gamma} = \begin{pmatrix} (F_3 W_2 - F_2 W_3)/\cos L & (W_1 - F_1 \sin L)/\cos L & F_1 \\ (F_1 W_3 - F_3 W_1)/\cos L & (W_2 - F_2 \sin L)/\cos L & F_2 \\ (F_2 W_1 - F_1 W_2)/\cos L & (W_3 - F_3 \sin L)/\cos L & F_3 \end{pmatrix} \quad (15)$$

APPENDIX B

Equations for Strapdown Fine Alignment

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Equations for Strapdown Fine Alignment

The six measurements used for alignment, three from the accelerometers and three from the gyros, may be expressed as

$$\underline{z} = \begin{pmatrix} \underline{f}^{p} \\ \\ \underline{\omega}^{p}_{ip} \end{pmatrix} = \begin{pmatrix} C^{p}_{n}\,\underline{f}^{n} \\ \\ C^{p}_{n}\,\underline{\omega}^{n}_{ip} \end{pmatrix} + \underline{v}' \qquad (1)$$

where

$\underline{f}^{p}$ = accelerometer outputs in platform coordinates

$\underline{\omega}^{p}_{ip}$ = gyro outputs in platform coordinates

C^{p}_{n} = direction cosine matrix which transforms vectors from east-north-up navigation coordinates to forward-leftwing-up platform coordinates

$\underline{v}'$ = gaussian white measurement noise vector with with covaraince matrix R

The inputs to the accelerometers and gyros, $\underline{f}^{n}$ and $\underline{\omega}^{n}_{ip}$ respectively, are given as

$$\begin{aligned} \underline{f}^{n} &= (\,0 \quad 0 \quad g\,)^{T} \\ \underline{\omega}^{n}_{ip} &= (\,0 \quad \Omega\cos L \quad \Omega\sin L\,)^{T} \end{aligned} \qquad (2)$$

where g is gravity, Ω is earth rate and L is the geographic latitude.

Expanding the measurement vector $\underline{z}$ about current indicated values, $\underline{z}^I$, to first order yields

$$\delta \underline{z} = \underline{z} - \underline{z}^I = \begin{pmatrix} \delta C_n^p \underline{f}^n \\ \\ \delta C_n^p \underline{w}_{ip}^n \end{pmatrix}_I + \begin{pmatrix} C_n^p \delta \underline{f}^n \\ \\ C_n^p \delta \underline{w}_{ip}^n \end{pmatrix} + \begin{pmatrix} \underline{v}'_f \\ \\ \underline{v}'_\omega \end{pmatrix} \tag{3}$$

where $\underline{v}$ and $\underline{v}$ are gaussian white measurement noise vectors associated with the accelerometers and gyros respectively. To first order

$$\delta C_n^p = -C_n^p (\delta \underline{\theta}_{np}^n \times) \tag{4}$$

where $\delta\underline{\theta}_{np}$ is a vector of platform tilt errors with respect to the navigation frame. Therefore, equation (3) may be written as

$$\delta \underline{z} = \underline{z} - \underline{z}^I = H\underline{x} + \underline{v}'^p \tag{5}$$

where the state vector

$$\underline{x} = \delta \underline{\theta}_{np} \tag{6}$$

and the measurement matrix

$$H = \begin{pmatrix} C_n^p (\underline{f}^n x) \\ \\ C_n^p (\underline{\omega}_{ip}^n x) \end{pmatrix} \tag{7}$$

In (5), the six dimensional noise vector

$$\underline{v}'^p = \begin{pmatrix} \underline{v}'^p_f \\ \\ \underline{v}'^p_\omega \end{pmatrix} = \begin{pmatrix} C_n^p \, \underline{f}^n + \underline{v}_f^p \\ \\ C_n^p \, \underline{\omega}_{ip}^n + \underline{v}_\omega^p \end{pmatrix} \tag{8}$$

is composed of sensor noise $\underline{v}_f$ and $\underline{v}_\omega$ plus the uncertainty associated with the inputs $\underline{f}^n$ and $\underline{\omega}_{ip}^n$, equations (2), expressed in platform coordinates.

In general, the propagation of the state vector $\underline{x}$ over time is given as

$$\underline{x}_k = \Phi_k \underline{x}_{k-1} + \underline{w}_k \tag{9}$$

where Φ_k is a known 3x3 transition matrix between t_{k-1} and t_k, and $\underline{w}_k$ is a random disturbance vector. In the present application, $\underline{w}_k = \underline{0}$ and Φ_k is the identity matrix. Hence there is no loss of information between measurement updates. After a measurement update, the covariance of $\underline{x}$ is given as

$$P_k^+ = (I - KH_k)P_k^- \tag{10}$$

where P is the covariance prior to the measurement, and K is the Kalman gain matrix, given as

$$K = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{11}$$

The state vector will be updated in accordance with

$$x_k = x_{k-1} + K z_{RES} \tag{12}$$

where the measurement residual

$$z_{RES} = \delta z - \delta \hat{z} \tag{13}$$

and K is the optimal Kalman gain matrix. Since, from (5)

$$\delta z = z - z^I$$

then the measurement residual is formed as

$$z_{RES} = z - z^I - (\hat{z} - z^I)$$

or

$$z_{RES} = z - \hat{z} \tag{14}$$

The predicted measurement value, $\hat{z}$, in (14) is obtained by using (1). Thus

$$\hat{\underline{z}} = \begin{pmatrix} \hat{C}_n^p \underline{f}^n \\ \\ \hat{C}_n^p \underline{\omega}_{ip}^n \end{pmatrix} \tag{15}$$

where $\hat{C}_n^p$ is the updated value of the alignment matrix given in terms of the updated state variable, $\underline{x} = \delta\underline{\theta}_{np}^n$, as

$$\begin{aligned} \hat{C}_n^p(k) &= \hat{C}_n^p(k-1)\left[I - (\delta\underline{\theta}_{np}^n x) + \frac{1}{2!}(\delta\underline{\theta}_{np}^n x)^2 - \dots\right] \\ &= \hat{C}_n^p(k-1)\exp\left[-(\delta\underline{\theta}_{np}^n x)\right] \end{aligned} \tag{16}$$

In order to effect a measurement update, it is necessary to provide the covariance of the noise expressed in equation (8). To obtain this requires evaluating the 6x6 matrix

$$R = \left(\begin{array}{c|c} E\left\{\underline{v}_f' \, \underline{v}_f'^T\right\} & 0 \\ \hline 0 & E\left\{\underline{v}_\omega' \, \underline{v}_\omega'^T\right\} \end{array}\right) \tag{17}$$

where $\underline{v}_f'$ and $\underline{v}_\omega'$ are given by equation (8) as

$$\underline{v}_f'^p = C_n^p \delta\underline{f}^n + \underline{v}_f^p \tag{18}$$

and

$$\underline{v}_\omega'^p = C_n^p \delta\underline{\omega}_{ip}^n + \underline{v}_\omega^p \tag{19}$$

We assume here that during alignment the platform will be maintained nearly vertical and that platform forward will be approximately pointed to the north, east, south or west.

Considering first equation (15), we obtain

$$E\{\underline{v}_f' \underline{v}_f'^T\} = C_n^p E\{\delta \underline{f}^n \delta \underline{f}^{n^T}\} C + E\{\underline{v}_f^p \underline{v}_f^{p^T}\} \tag{20}$$

where $E\{\underline{v}_f^p \underline{v}_f^{pT}\}$ is the covariance of the accelerometer noise, i.e.

$$E\{\underline{v}_f^p \; \underline{v}_f^{p^T}\} = \begin{pmatrix} \sigma_{f1}^2 & 0 & 0 \\ 0 & \sigma_{f2}^2 & 0 \\ 0 & 0 & \sigma_{f3}^2 \end{pmatrix} \tag{21}$$

using

$$f_u = g_u = -g_o\left[1 + .005279 \sin^2 L - \frac{2h}{Re}\right] \tag{22}$$

where

g_o = sea level gravity

L = geographic latitude

h = altitude

Re = equatorial earth radians

then

$$\delta f_u = -a_1 \delta s + a_2 \delta h + a_3 \delta g_o \tag{23}$$

where

$$a_1 = .005279\left(\frac{g_o}{Re}\right) \sin 2L \tag{24}$$

$$a_2 = \frac{2g_o}{Re} \tag{25}$$

$$a_3 = 1 + .05279 \sin^2 L - \frac{2h}{Re} \tag{26}$$

In the above expressions

δs = north-south location error ($=R_e dL$)(feet)

δh = altitude error (feet)

δg_o = error in gravity (ft/sec^2)

Using equation (23), the first term in equation (20) becomes

$$C_n^p E\left\{\underline{v}_f' \underline{v}_f'^T\right\} C_p^n = a_1^2 E\left\{\delta s^2\right\} + a_2^2 E\left\{\delta h^2\right\} + a_3^2 E\left\{\delta g_o^2\right\} \tag{27}$$

The first three (diagonal) elements of (17) become

$$\begin{aligned} R(1) &= \sigma_{f1}^2 \\ R(2) &= \sigma_{f2}^2 \\ R(3) &= \sigma_{f3}^2 + a_1^2 E\left\{\delta s^2\right\} + a_2^2 E\left\{\delta h^2\right\} + a_3^2 E\left\{\delta g_c^2\right\} \end{aligned} \tag{28}$$

In a similar fashion, for the gyros we have

$$E\left\{v_\omega' v_\omega'^T\right\} = C_n^p E\left\{\delta \underline{\omega}_{ip}^n \delta \underline{\omega}_{ip}^{nT}\right\} C_p^n + E\left\{\underline{v}_\omega^p \underline{v}_\omega^{pT}\right\} \tag{29}$$

where

$$E\left\{\underline{v}_\omega^p \underline{v}_\omega^{pT}\right\} = \begin{pmatrix} \sigma_{\omega 1}^2 & 0 & 0 \\ 0 & \sigma_{\omega 2}^2 & 0 \\ 0 & 0 & \sigma_{\omega 3}^2 \end{pmatrix} \tag{30}$$

is the covariance of the gyro instrument noise.

From equation (2) we have

$$\delta \underline{\omega}_{i\beta}^{n} = (\,0 \quad -\left(\frac{\Omega}{R_e}\right)\sin L \quad \left(\frac{\Omega}{R_e}\right)\cos L\,)^T \,\delta s. \tag{31}$$

Calculating $C_n^{\beta} E\left\{\delta \underline{\omega}_{i\beta}^{n}\, \delta \underline{\omega}_{i\beta}^{n\,T}\right\} C_{\beta}^{n}$ using the previously noted assumptions and combining with (30) gives the remaining diagonal elements of the measurement matrix

$$\begin{aligned} R(4) &= C_{12}^2\, b_1^2 E\{\delta s^2\} + \sigma_{\omega 1}^2 \\ R(5) &= C_{22}^2\, b_1^2 E\{\delta s^2\} + \sigma_{\omega 2}^2 \\ R(6) &= b_2 E\{\delta s^2\} + \sigma_{\omega 3}^2 \end{aligned} \tag{32}$$

where

$$\begin{aligned} b_1 &= \left(\frac{\Omega}{Re}\sin L\right)^2 \\ b_2 &= \left(\frac{\Omega}{Re}\cos L\right)^2 \end{aligned} \tag{33}$$

REFERENCES

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